Trinity Western University
Department of Mathematical Sciences
MATH 250 (Linear Algebra)
Sample Mid-Term Exam I

1. Show that the matrix

$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$$

satisfies the equation

$$(A + 3I)^2(A - I) = 0$$

Use the above equation to prove that A is invertible and compute A^{-1}.

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 5 \\ 1 & -7 & 13 \end{pmatrix}$$

Show that A is not invertible by finding a lower-triangular matrix L such that $A = LU$, where U is an upper-triangular matrix which has at least one row of zeros.

3. Find for what values of c the following matrix is not invertible. Find the inverse of the matrix for the remaining values of c.

$$\begin{pmatrix} 1 & 0 & -c \\ -1 & 3 & 1 \\ 0 & 2c & -4 \end{pmatrix}$$

4. Using Cramer's rule solve the following system of equations for z:

$$\begin{aligned} x + y + z + w &= 10 \\ x + 2y + 3z + 4w &= 30 \\ x + 4y + 9z + 16w &= 100 \\ x + 8y + 27z + 64w &= 354 \end{aligned}$$

5. Assume that there are three classes – upper U, middle M, and lower L – and that social mobility is modeled as follows:

i) Of children of U parents, 70% remain U while 20% become L and 10% become M.

ii) Of children of M parents, 80% remain M while 10% become L and 10% become U.

iii) Of children of L parents, 60% remain L while 10% become U and 30% become M.

Find the probability that the grandchild of L parents becomes U. Also find the long-term breakdown of society into classes.