

# §14.7-14.8, Py ch20: Binary Trees

- **Quiz09** today

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CMPT14x  
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# Review last time (§12.8-12.12)

## ■ Linked lists

- Type definition, creating a new list
  - ◆ Inserting in nth position
  - ◆ Insert at head, append to tail
  - ◆ Deleting
- Algorithmic efficiency
- Circularly linked lists
- Bidirectional lists

# Quiz09: 10 minutes

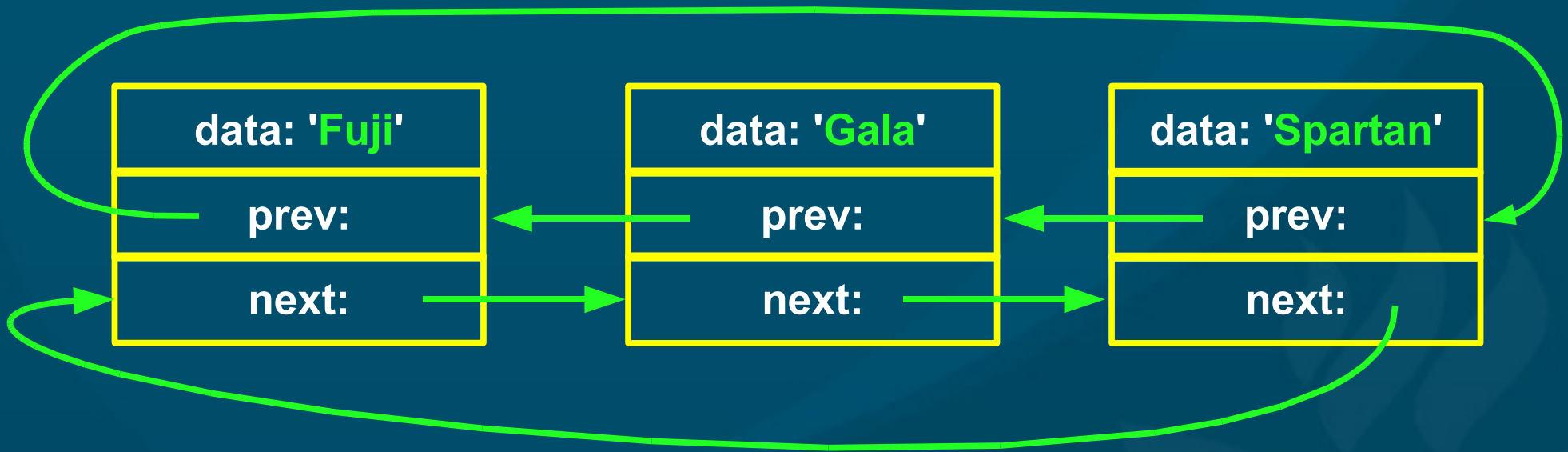
- Let `numApples` be an `integer` variable, and let `numApplesPtr` be a `pointer` to `numApples`.
  - Describe the `contrast` between the value in `numApples` and the value in `numApplesPtr`. [5]
  - Write `C` code equivalent to this Python code:
    - ◆ `numApples += 1`  
but without using `numApples` directly! [3]
  - Do the same in `M2`. [3]
- Draw a `diagram` representing a `circular doubly-linked list` with three elements: `'Fuji'`, `'Gala'`, `'Spartan'`. Clearly label all pointers. [9]

# Quiz09 answers: #1

- Let `numApples` be an integer variable, and let `numApplesPtr` be a pointer to `numApples`.
  - Describe the contrast between the value in `numApples` and the value in `numApplesPtr`.
  - The value in `numApples` is an integer representing, e.g., the number of apples I own. The value in `numApplesPtr` is an address in memory of where `numApples` is stored.
- C code:
  - ◆ `*numApplesPtr = (*numApplesPtr) + 1;`
- M2 code:
  - ◆ `^numApplesPtr := (^numApplesPtr) + 1;`

# Quiz09 answers: #2

- Draw a **diagram** representing a **circular doubly-linked list** with three elements: 'Fuji', 'Gala', 'Spartan'. Clearly label all pointers.

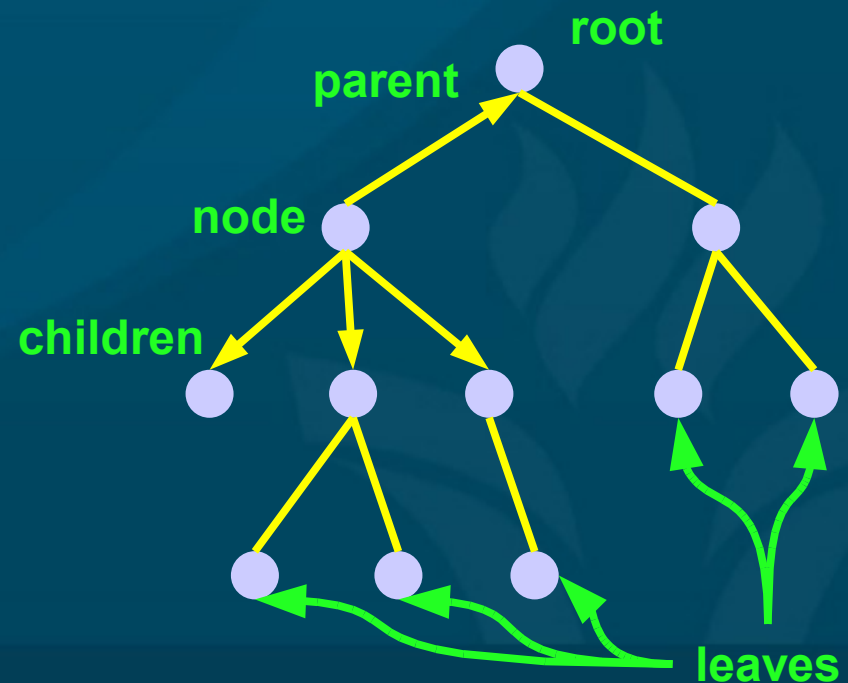


# What's on for today

- Trees:
  - Definition of terms:
    - ◆ Parent, children, root, leaves, degree, depth, level, forest
  - Depth-first vs. breadth-first search
  - Binary trees: pre/in/post-order traversal
  - Binary search trees (BST):
    - ◆ Type definition
    - ◆ Search, Insert, Delete
    - ◆ Algorithmic efficiency of BST Search

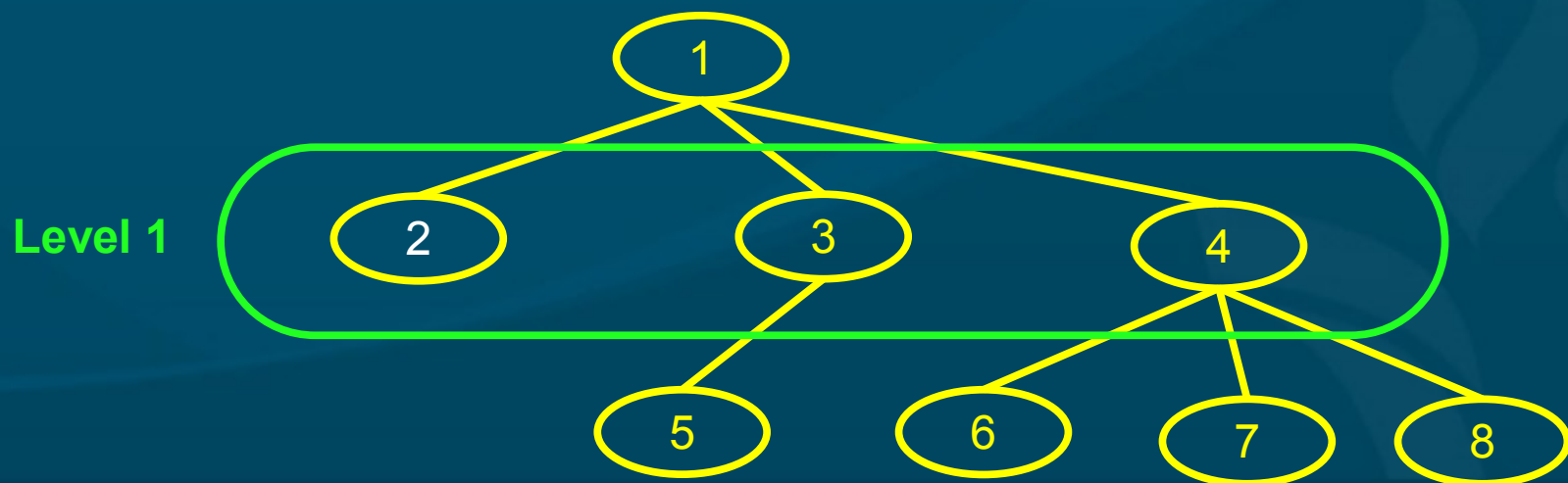
# Trees

- Another kind of dynamic ADT is the **tree**:
  - **Root**: starting node (one per tree)
    - ◆ Could also have a **forest** of several trees
  - Each node has at most one **parent**, and zero or more **children**
  - **Leaves**: no children
  - **Depth**: length of longest path from root
  - **Degree**: max # of children per node



# Searching trees

- A **depth-first** search of a tree pursues each path down to a leaf, then **backtracks** to the next path
  - ◆ 1-2      1-3-5      1-4-6      4-7      4-8
- A **breadth-first** search finishes each **level** before moving on to the next:
  - ◆ 1    2-3-4      5-6-7-8





# Binary search trees

- Binary trees (degree=2) are handy for keeping things in sorted order:

```
class BST:
```

```
    def __init__(self, data=None):
```

```
        self.data = data
```

```
        self.left = None
```

```
        self.right = None
```

```
        (* could also have a parent ptr *)
```

```
root = BST( 'Braeburn' )
```

```
root.left = BST( 'Ambrosia' )
```

```
root.right = BST( 'Gala' )
```

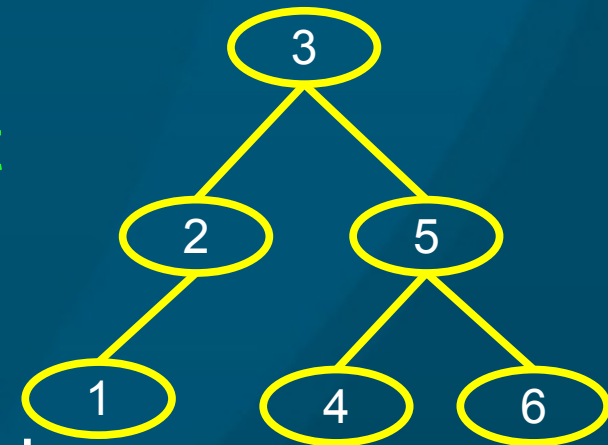
```
root.right.left = BST( 'Fuji' )
```



- Everything in left subtree is smaller
- Everything in right subtree is bigger

# Binary tree traversals

- **Pre-order** traversal of binary tree:
  - Do **self** first, then **left** child, then **right**
    - ◆ 3 – 2 – 1 – 5 – 4 – 6
- **In-order** traversal:
  - Do **left** child, then **self**, then **right** child
    - ◆ 1 – 2 – 3 – 4 – 5 – 6 (**sorted** order in BST)
    - ◆ e.g. expressions: “12 + (2 \* 5)”
- **Post-order** traversal:
  - Do **both** children first before **self**
    - ◆ 1 – 2 – 4 – 6 – 5 – 3
    - ◆ e.g. Reverse Polish Notation: 12, 2, 5, \*, +



# Searching a BST

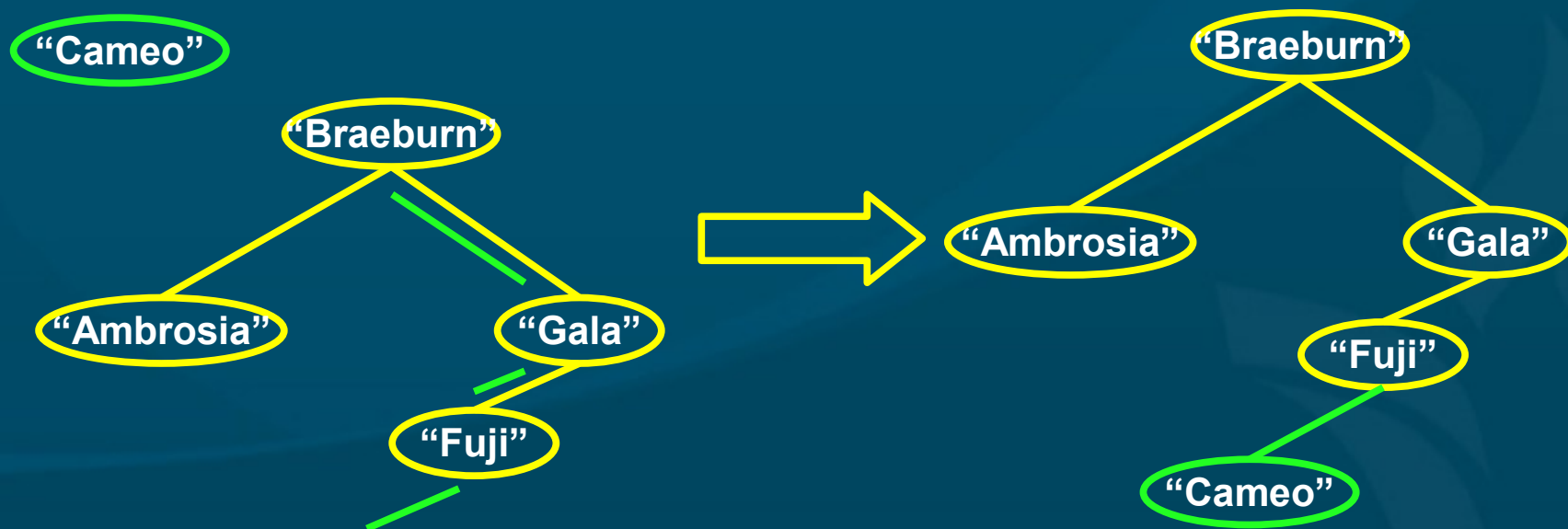
- Recursive algorithm:

```
def search (self, key):  
    if key == self.data:  
        return self  
    elif key < self.data and self.left != None:  
        return self.left.search(key)  
    elif key > self.data and self.right != None:  
        return self.right.search(key)  
    else:  
        return None
```



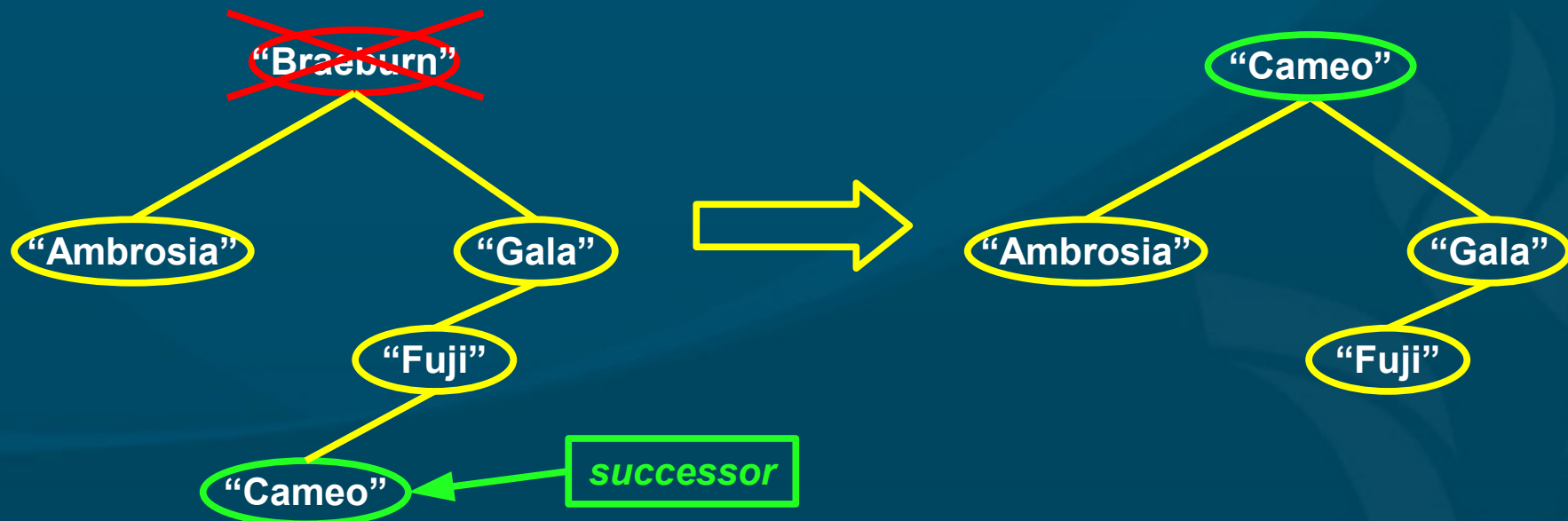
# Inserting into a BST

- Keep it sorted: insert in a **proper** place
- One choice: always insert as a **leaf**
  - Use `search()` algorithm to hunt for where the node ought to be if it were already in the tree



# Deleting from a BST

- Need to **maintain** sorted structure of BST
- Replace node with **predecessor** or **successor** leaf
  - Predecessor: **largest** node in **left** subtree
  - Successor: **smallest** node in **right** subtree



# BSTs and algorithmic efficiency

- Searching in a **balanced** binary search tree takes worst-case  $O(\log n)$  running time:
  - **Depth** of balanced tree is  $\log_2 n$
  - Compare with **arrays/linked lists**:  $O(n)$
- But depending on order of inserts, tree may be **unbalanced**:
  - ◆ Insert in **order**: Ambrosia, Braeburn, Fuji, Gala:
  - ◆ Tree **degenerates** to linked-list
  - ◆ Searching becomes  $O(n)$
- Keeping a BST **balanced** is a larger topic
  - ◆ e.g., **Splay-trees**



# Review of today

- Trees:
  - Definition of terms:
    - ◆ Parent, children, root, leaves, degree, depth, level, forest
  - Depth-first vs. breadth-first search
  - Binary trees: pre/in/post-order traversal
  - Binary search trees (BST):
    - ◆ Type definition
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    - ◆ Algorithmic efficiency of BST Search

# TODO

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- **Lab10** due next week:
  - Due date postponed for all lab sections
  - No Lab11
  - Implement one of your old labs 2-7 in M2
  - Full lab-writeup (may reuse old writeup)
- **HW11** due Fri:
  - delete() for doubly-linked list
- **Paper** due next Wed 6Dec