

# §4.8-4.10, Py ch5-6: Recursion

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CMPT14x

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# Review from last time (ch4)

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- Some **debugging** tips
- A fun example: **ROT13**
  - **ord()**, **chr()**, string indexing, **len()**
  - **Stub** program

# Addendum: iterating a string

- Iterating through a string:

```
for idx in range(len(myString)):
```

```
    myChar = myString[idx]
```

- Shorthand in Python:

(can treat strings as **lists** of characters)

```
for myChar in myString:
```

```
    myChar ...
```

- For example:

```
for myChar in "Hello World!":
```

```
    print myChar
```

# What's on for today (§4.8, Py ch5-6)

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- Recursive functions
  - Factorial example
- Call stack, backtrace
  - Fibonacci example
- Abstract Data Types
  - Type hierarchy
- Enumerations

# Recursion

- **Recursion** is when a function invokes itself
- Classic example: **factorial** (!)
  - $n! = n(n-1)(n-2)(n-3) \dots (3)(2)(1)$
  - $0! = 1$
- Compute **recursively**:
  - **Inductive step**:  $n! = n * (n-1)!$
  - **Base case**:  $0! = 1$
- Inductive step: **assume**  $(n-1)!$  is calculated correctly; then we can find  $n!$
- Base case is needed to tell us where to **start**

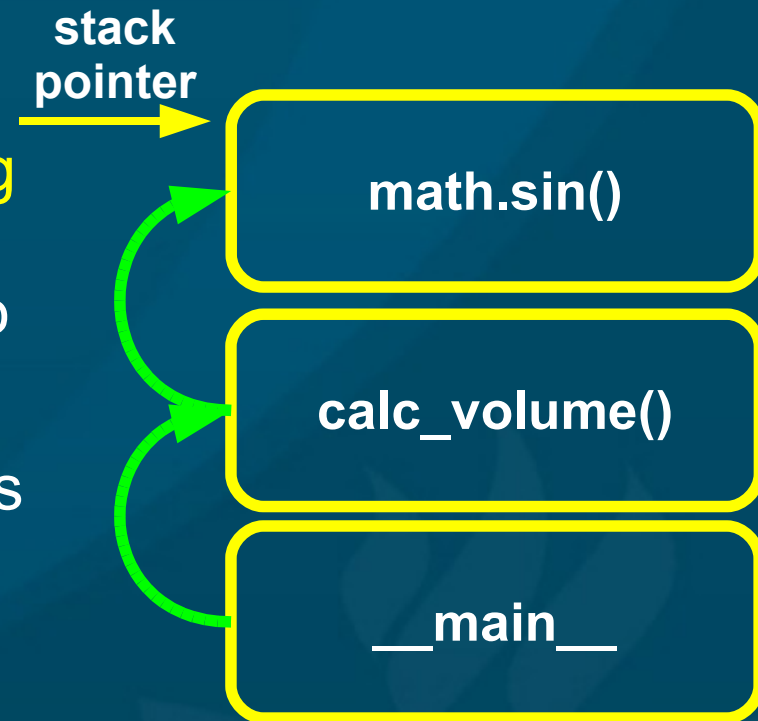
# factorial() in Python

```
def factorial(n):  
    """Calculate n!. n should be a positive integer."""  
    if n == 0:                # base case  
        return 1  
    else:                     # inductive step  
        return n * factorial(n-1)
```

- **Progress** is made each time: `factorial(n-1)`
- Base case prevents **infinite** recursion
- What about `factorial(-1)`? Or `factorial(2.5)`?

# The call stack

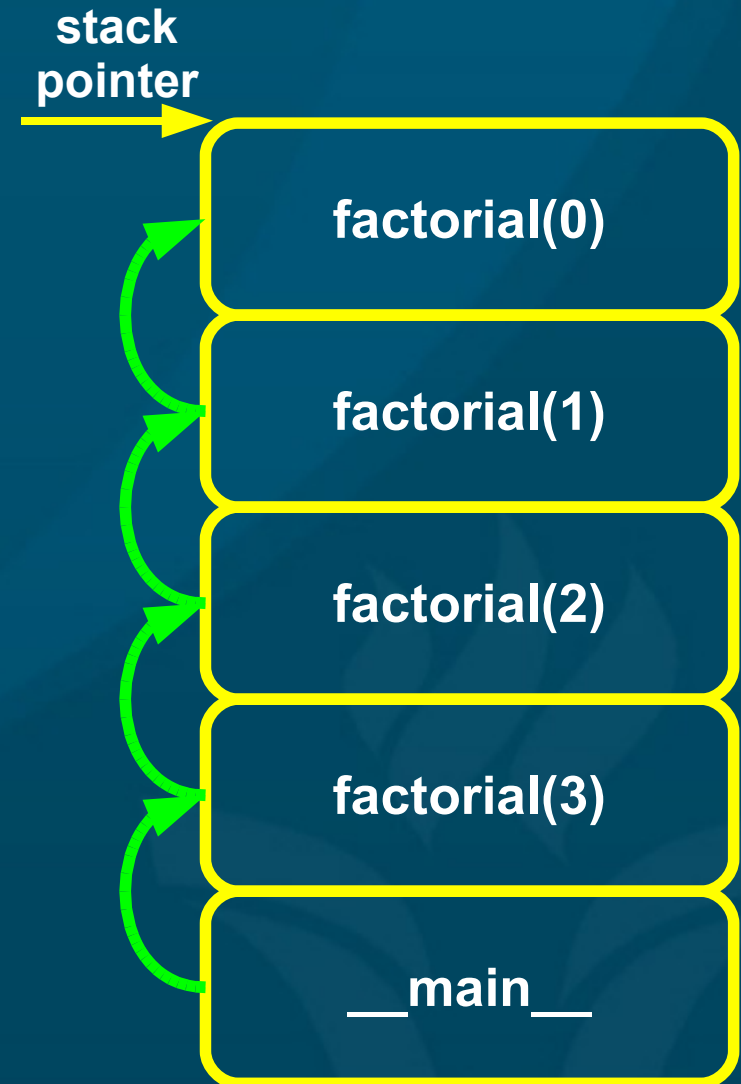
- When a program is running, an area of **memory** is set aside to store local **variables**, the state of the program, etc.
- When a **procedure** is invoked, the **calling context** is saved, and a new chunk of memory is allocated for the procedure to use: its **stack frame**
- When the procedure finishes, its frame is **released** and control goes back to the calling context
- The **stack pointer** keeps track of what frame is currently running



# Call stack for recursive functions

```
def factorial(n):  
    """Compute the factorial of a  
    positive integer."""  
    if n == 0:  
        return 1  
    else:  
        return n*factorial(n-1)
```

- If there were any **local** variables, each frame would have its own instance of the local variables
- When an error (exception) happens, IDLE shows a **backtrace**: part of the call stack





# Another recursive ex.: Fibonacci

- **Fibonacci** sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34,...

- Each number is the **sum** of the two previous

**def fibonacci(n):**

```
    """Compute the n-th Fibonacci number.
```

```
    pre: n should be a positive integer.
```

```
    """
```

```
    if n == 0 or n == 1:
```

```
        # base case
```

```
        return 1
```

```
    else:
```

```
        # inductive step
```

```
        return fibonacci(n-2) + fibonacci(n-1)
```

- Note: very **inefficient** algorithm!

# Abstract Data Types

- Recall the categorization of
  - **Atomic** vs. **Aggregate** (compound) types
- Some examples of **atomic** data types:
  - Real (**float**), integer (**int**), Boolean (**bool**)
  - Character (if the language has such a type)
- Some examples of **aggregate** data types:
  - Arrays, tuples, dictionaries, records/structs
- **Abstract Data Type** (ADT):
  - Details of **implementation** are hidden from user (how to represent a float in binary form?)

# M2 type hierarchy (partial)

- **Atomic** types
  - Scalar types
    - ◆ Real types (REAL, LONGREAL)
    - ◆ Ordinal types (CHAR)
      - Whole number types (INTEGER, CARDINAL)
      - Enumerations (§5.2.1) (BOOLEAN)
      - Subranges (§5.2.2)
- Structured (**aggregate**) types
  - Arrays (§5.3)
    - ◆ Strings (§5.3.1)
  - Sets (§9.2-9.6)
  - Records (§9.7-9.12)
- Also can have **user-defined** types

# Python type hierarchy (partial)

## ■ Atomic types

### ● Numbers

- ◆ Integers (int, long, bool): `5`, `500000L`, `True`
- ◆ Reals (float) (only double-precision): `5.0`
- ◆ Complex numbers (complex): `5+2j`

## ■ Container (**aggregate**) types

### ● Immutable sequences

- ◆ Strings (str): `"Hello"`
- ◆ Tuples (tuple): `(2, 5.0, "hi")`

### ● Mutable sequences

- ◆ Lists (list): `[2, 5.0, "hi"]`

### ● Mappings

- ◆ Dictionaries (dict): `{"apple": 5, "orange": 8}`

# Enumeration types in M2 (also C)

## TYPE

```
DayName = (Sun, Mon, Tue, Wed, Thu, Fri, Sat);
```

## VAR

```
today : DayName;
```

## BEGIN

```
today := Mon;
```

- We could have used **CARDINALs** instead (and indeed the underlying implementation does)
  - But the logical semantic of today's type is a **DayName** type, not a **CARDINAL**
- Can be thought of as Sun=0, Mon=1, Tue=2, ...
- No explicit enumeration scheme in Python

# Review of today (§4.8, Py ch5-6)

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- Recursive functions
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- Call stack, backtrace
  - Fibonacci example
- Abstract Data Types
  - Type hierarchy
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# TODO

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- Lab 02 due tonight:
  - M2 ch3 # (16 / 17 / 23a / 23b / 23c)
- Quiz03 (ch4) this Fri
- Lab 03 due next Wed:
  - M2 ch4 # (23 / 27 / 36)
- HW03 due next Mon: 4.11 #7, 18; 5.11 # 15
- Read through M2 ch5 and Py ch7, plus Py ch10
  
- Midterm ch1-5 next week Fri 5Oct