Transform Matrices

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CMPT370
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Review last time

- Scalars, vectors, points
- Vector spaces, affine spaces (+point)
- Lines, rays, line segments
- Curves, surfaces
- Normal vectors
- Convex hull
- Linear independence
- Basis, frame (+point)

Homogeneous coordinates

- We use a 4-tuple as a homogeneous representation for both points and vectors
  - $[\ \alpha_1\ \alpha_2\ \alpha_3\ 0\ ]^T$ is a vector
  - $[\ \beta_1\ \beta_2\ \beta_3\ 1\ ]^T$ is a point
  - Depends on current coordinate frame
  - Any 4-tuple $[\ x\ y\ z\ w\ ]^T$ maps to a point via
    - $[\ x/w\ y/w\ z/w\ 1\ ]^T$
    - If w=0, the 4-tuple represents a vector
  - Each point in 3D maps to a line through the origin in 4D

Changing coordinate systems

- Say we have a vector whose representation in one basis (e_1, e_2, e_3) is $v = \{\alpha_1\ \alpha_2\ \alpha_3\}$.
  - What is the representation for the same vector in a different basis, $\{d_1, d_2, d_3\}$?
- Represent each old basis vec e_i in the new basis:
  - $e_1 = \gamma_{11} d_1 + \gamma_{12} d_2 + \gamma_{13} d_3$
  - $e_2 = \gamma_{21} d_1 + \gamma_{22} d_2 + \gamma_{23} d_3$
  - $e_3 = \gamma_{31} d_1 + \gamma_{32} d_2 + \gamma_{33} d_3$

3x3 Transform matrix

- These nine coefficients form a 3x3 vector transform matrix M:
  - $$M = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix}$$
  - $w = M^T v$, where
    - $v = \{\alpha_1\ \alpha_2\ \alpha_3\}$ is the representation in the old basis
    - $w = \{\beta_1\ \beta_2\ \beta_3\}$ is the representation in the new basis

Change of frames

- Something similar happens to change frames:
  - Old frame is (P, e_1, e_2, e_3)
  - New frame is (Q, d_1, d_2, d_3)
  - Represent old frame in new basis
  - 12 degrees of freedom in affine transform

$$M = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & 0 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & 0 \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & 0 \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & 1 \end{pmatrix}$$

Translation matrix

$$T = \begin{pmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Translate a point p by multiplying by T:
  - p' = Tp

Scaling matrix

$$S = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Scale a point p by multiplying by T:
  - p' = Tp
- Fixed point of origin (scaling away from origin)
- Reflection is via negative scale factors

Rotation matrix

$$R = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Rotate an angle theta about the z axis
- Similar matrices for rotating about x, y axes
- 3 Euler angles
- Order of operations is important!
  - Rotation in 3D is non-Abelian

TODO

- Lab3 due this Thu 22Feb
  - OpenGL 3D model viewer
  - Mouse interaction: transl, rot, scale
  - Show rendering speed in polygons/sec
  - You may use CubeView as a base