§12.7-12.11, 14.7-14.8: Linked Lists and Binary Search Trees

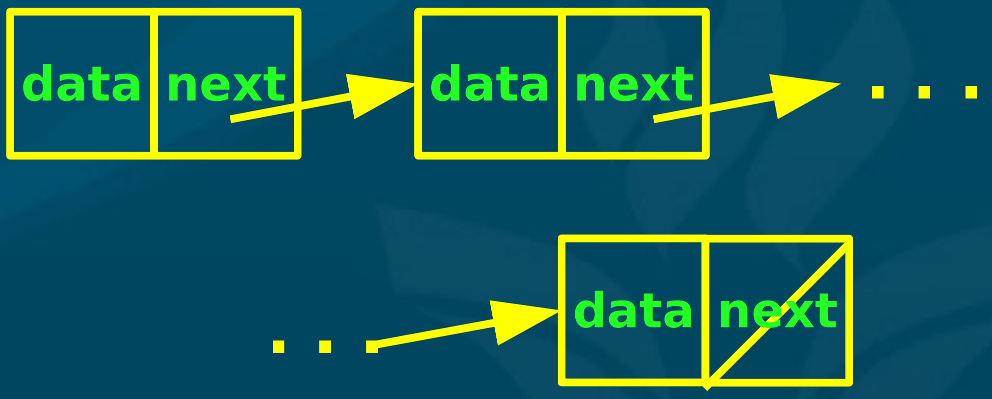
26 Nov 2008
CMPT14x
Dr. Sean Ho
Trinity Western University

Linked lists: creating

- A linked list is a dynamic ADT where each item in the list contains a pointer to the next item:

```
class Node:
    def __init__(self, data=None, next=None):
        self.data = data
        self.next = next

n1 = Node()
n2 = Node()
n1.next = n2
```



Operations on linked lists

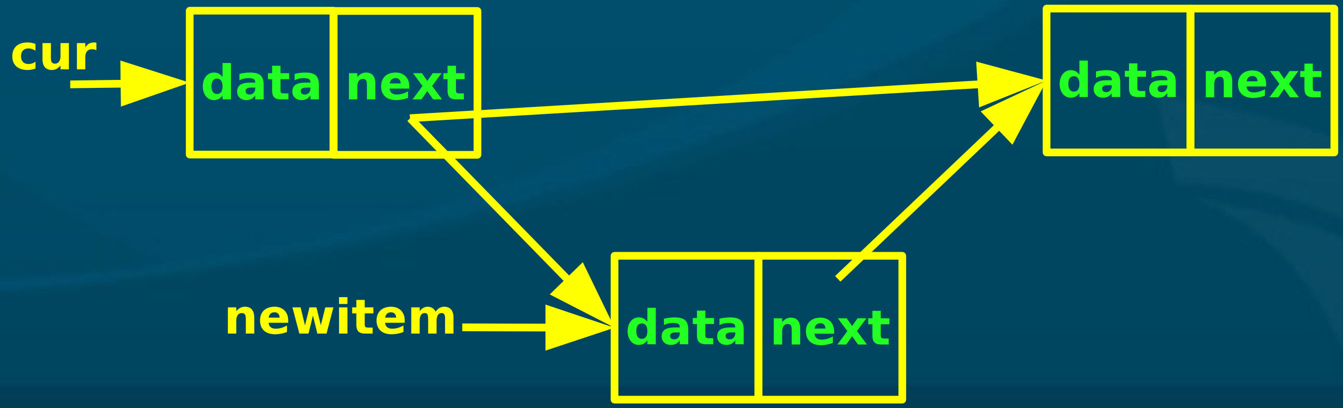
- Index into list (get a reference to n^{th} node)
- Print out the list
- Search list for given data (cargo/payload)
- Insert a new node into a linked list
- Delete a node from a linked list
  - By index (0, 1, 2, ...) or by cargo

Inserting a node into a linked list

- Follow pointers to get to the right spot
  - Create a new node with the given cargo
  - Thread new node into the list

```
newitem = Node(data)
newitem.next = cur.next
cur.next = newitem
```

  - What about inserting at head of list?



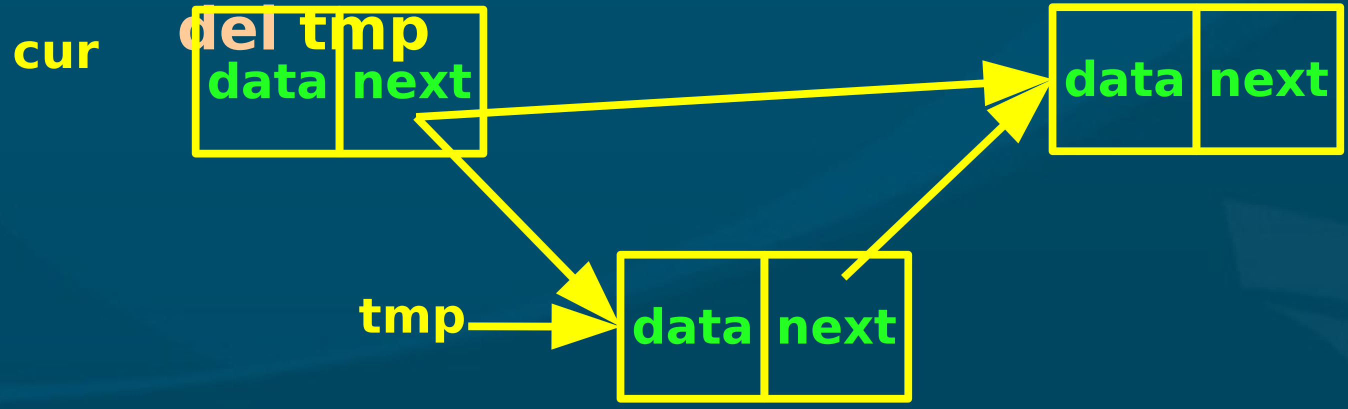
Insert() method: code

```
def insert (self, n, data=None):
    """Insert a new node into linked list at position n."""
    newitem = Node(data)
    if n == 0:                          # new head: modify self
        newitem.next = self
        self = newitem
    else:
        cur = self
        for idx in range(n-1):          # get to proper position
            cur = cur.next
        newitem.next = cur.next
        cur.next = newitem
```

Deleting from a linked list

- Follow pointers to find the item we want to delete
  - Sew up links to skip over the item
  - Deallocate the item from memory

```
tmp = cur.next
cur.next = tmp.next
del tmp
```



Linked lists: algorithmic efficiency

- Big-O notation: O(n) means # operations varies linearly with n
- For a linked list with n items:
  - Insert at head: don't have to traverse list: O(1)
  - Append to tail: must walk list: O(n)
  - General insert:
    - Worst-case: O(n)
    - Average-case: O(n/2), which is also O(n)
  - Deleting: also O(n)
- Double-headed list (keep a tail pointer):
  - Speeds up append-to-tail to O(1)

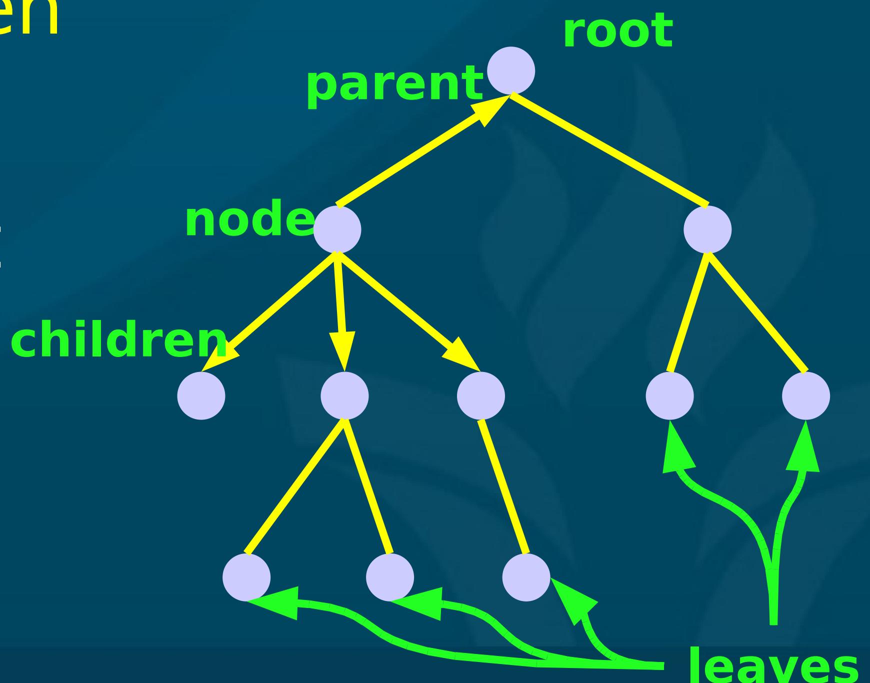
Variants of linked lists

- Circularly linked list:
  - tail.next = head
  - How to keep from infinite loop?
- Bidirectional linked list:

```
class Node:
    def __init__(self, data=None, prev=None,
      next=None):
        self.data = data
        self.prev = prev
        self.next = next
```

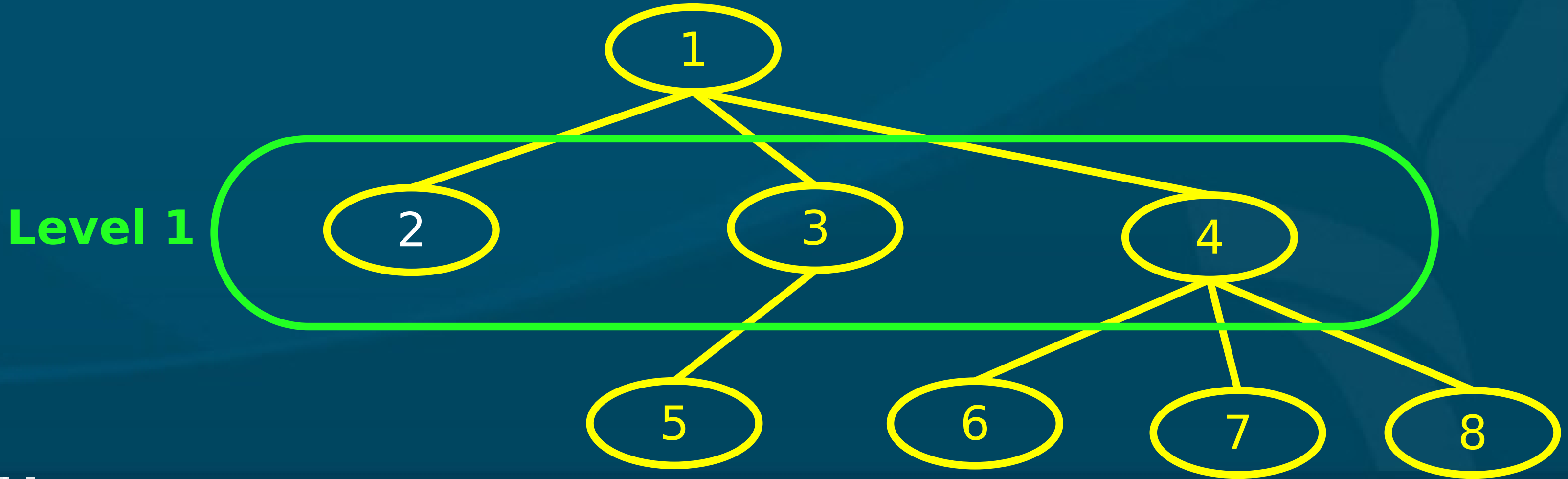
Trees

- Another kind of dynamic ADT is the tree:
  - Root: starting node (one per tree)
    - Could also have a forest of several trees
  - Each node has at most one parent, and zero or more children
  - Leaves: no children
  - Depth: length of longest path from root
  - Degree: max # of children per node



Searching trees

- A depth-first search of a tree pursues each path down to a leaf, then backtracks to the next path
  - 1-2 1-3-5 1-4-6 4-7 4-8
- A breadth-first search finishes each level before moving on to the next:
  - 1 2-3-4 5-6-7-8



Binary search trees

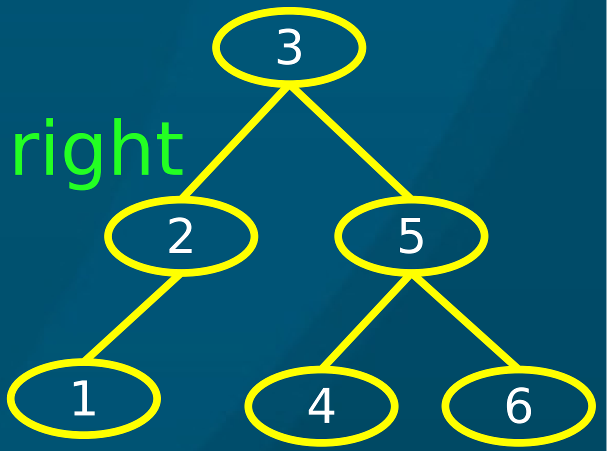
- Binary trees (degree=2) are handy for keeping things in sorted order:

```
class BST:
    def __init__(self, data=None):
        self.data = data
        self.left = None
        self.right = None
            (* could also have a parent ptr *)
root = BST( 'Braeburn' )
root.left = BST( 'Ambrosia' )
root.right = BST( 'Gala' )
root.right.left = BST( 'Fuji' )
```

- Everything in left subtree is smaller
- Everything in right subtree is bigger

Binary tree traversals

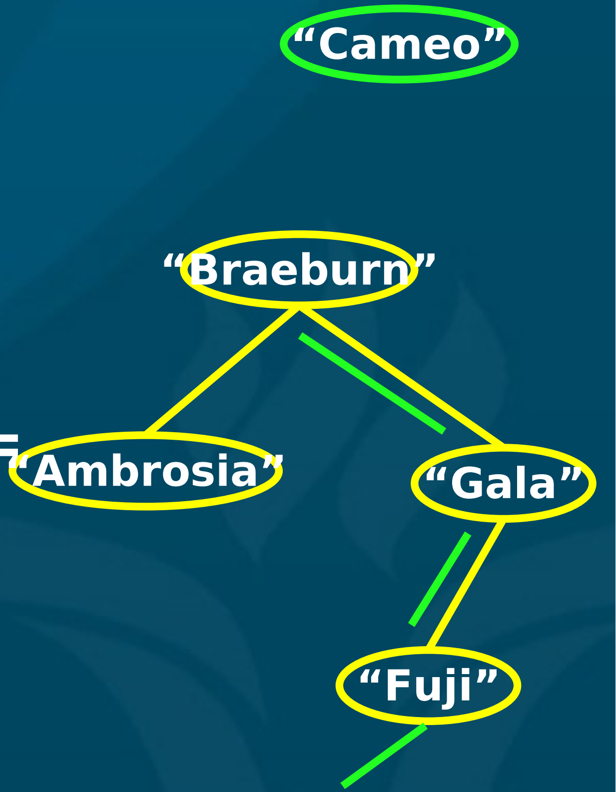
- Pre-order traversal of binary tree:
  - Do self first, then left child, then right
    - 3 – 2 – 1 – 5 – 4 - 6
- In-order traversal:
  - Do left child, then self, then right child
    - 1 – 2 – 3 – 4 – 5 – 6 (sorted order in BST)
    - e.g. expressions: "12 + (2 * 5)"
- Post-order traversal:
  - Do both children first before self
    - 1 – 2 – 4 – 6 – 5 - 3
    - e.g. Reverse Polish Notation: 12, 2, 5, *, +



Searching a BST

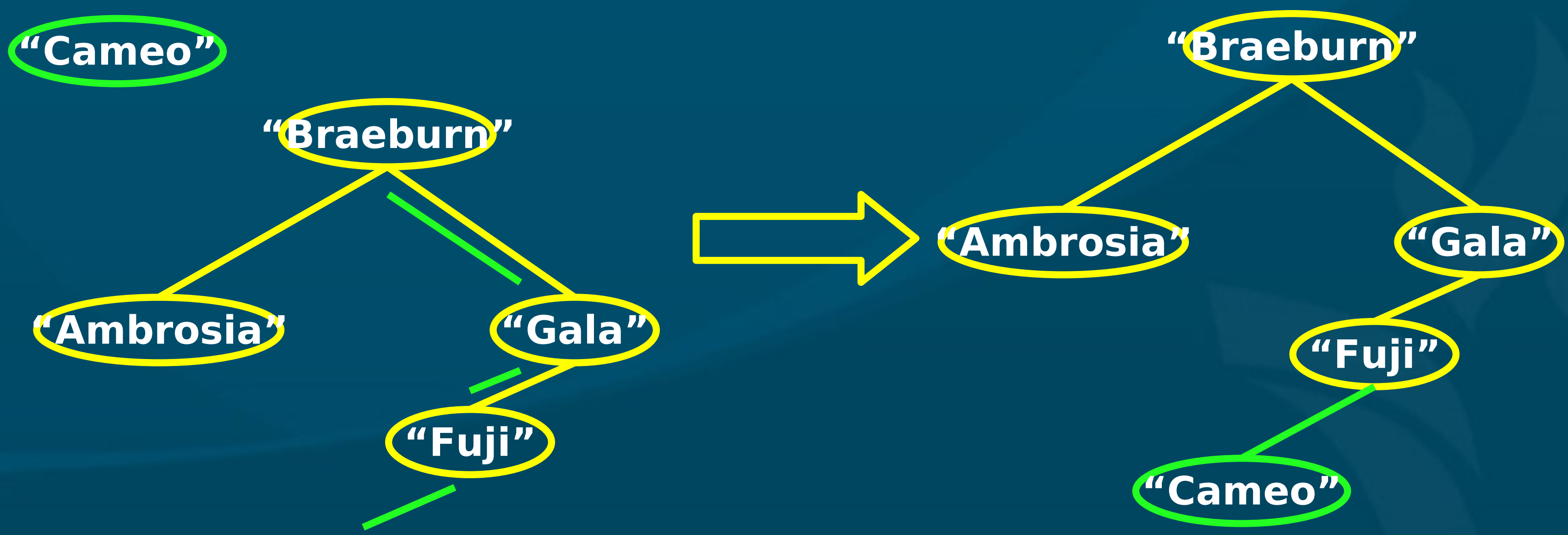
- Recursive algorithm:

```
def search (self, key):
    if key == self.data:
        return self
    elif key < self.data and self.left !=
      None:
        return self.left.search(key)
    elif key > self.data and self.right !=
      None:
        return self.right.search(key)
    else:
        return None
```



Inserting into a BST

- Keep it sorted: insert in a proper place
- One choice: always insert as a leaf
  - Use search() algorithm to hunt for where the node ought to be if it were already in the tree



Deleting from a BST

- Need to maintain sorted structure of BST
- Replace node with predecessor or successor leaf
  - Predecessor: largest node in left subtree
  - Successor: smallest node in right subtree

BSTs and algorithmic efficiency

- Searching in a balanced binary search tree takes worst-case O(log n) running time:
  - Depth of balanced tree is $\log_2 n$
  - Compare with arrays/linked lists: O(n)
- But depending on order of inserts, tree may be unbalanced:
  - Insert in order: Ambrosia, Braeburn, Fuji, Gala:
  - Tree degenerates to linked-list
  - Searching becomes O(n)
- Keeping a BST balanced is a larger topic
  - e.g., Splay-trees