

Recursion and Fractals

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CMPT166

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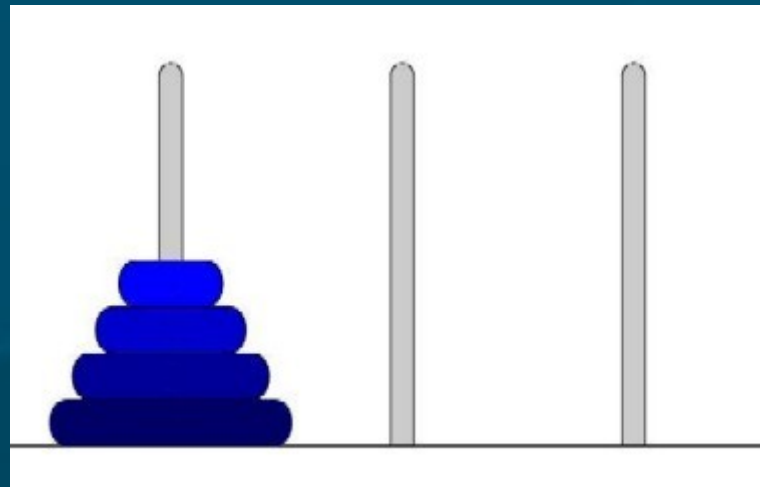
Trinity Western University

Review: Drawing in FLTK

- Complex shapes
 - Path: `vertex`, `curve`, `arc`
 - Transform matrix: `scale`, `rotate`, `translate`
- Clipping: `fl_push_clip()`, `fl_pop_clip()`
- Drawing images:
 - Direct: `fl_draw_image()`
 - With `Fl_Image` object:
 - ◆ `Fl_JPEG_Image` et al
 - ◆ `.draw()`

Recursion: Towers of Hanoi

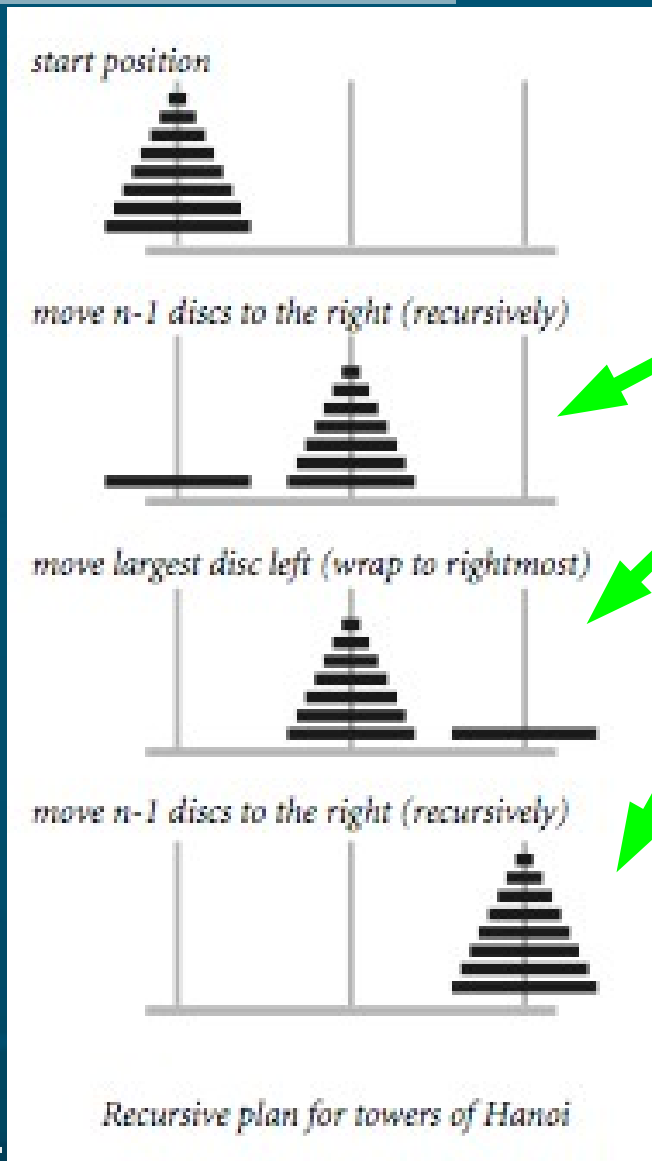
- The **problem**: N **disks** of increasing size are stacked on one of three **pegs**.
Move the stack to another peg.
 - You may only move the **top** disk on a peg
 - Only **smaller** disks may be on top of larger disks



Towers of Hanoi: the story

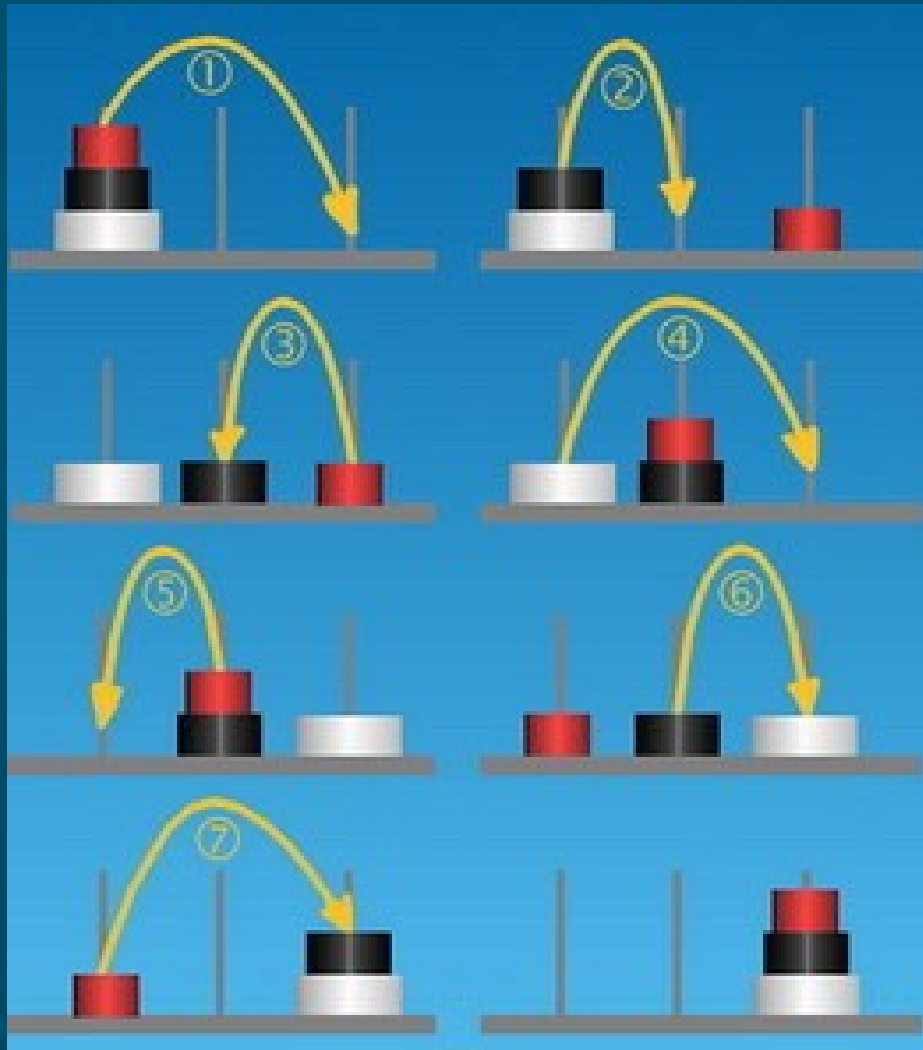
- "In the great temple at Benares, says he, beneath the **dome** which marks the centre of the world, rests a **brass plate** in which are fixed **three diamond needles**, each a cubit high and as thick as the body of a bee. On one of these needles, at the creation, God placed **sixty-four discs** of pure gold, the **largest** disc resting on the brass plate, and the others getting **smaller** and smaller up to the top one. This is the **Tower of Bramah**. Day and night unceasingly the priests transfer the discs from one diamond needle to another according to the fixed and immutable **laws** of Bramah, which require that the priest on duty must not move more than **one disc at a time** and that he must place this disc on a needle so that there is **no smaller disc below it**. When the sixty-four discs shall have been thus **transferred** from the needle on which at the creation God placed them **to one of the other needles**, tower, temple, and Bramahns alike will crumble into dust, and with a thunderclap **the world will vanish.**"

Towers of Hanoi: the solution



```
void hanoi(n, fr, to, tmp) {  
    if (n > 0) return;  
    hanoi(n-1, fr, tmp, to);  
    move(fr, to);  
    hanoi(n-1, tmp, to, fr);  
}
```

Towers of Hanoi: solution



- **Smallest** disk moves on odd turns, always in same direction
- Even turns have only **one** possibility
- Total number of moves is $2^n - 1$
- **64** disks @1 per sec: **140 million** years!

azerdark's Programmer's Kitchen

Gray codes

- Samuel Beckett, “Quad” play:
- Starting with an empty stage, characters **enter** and **exit** one at a time, but each subset of characters on the stage appears exactly **once**
 - **16** possible combinations
- **Gray code**: ordering such that only one **bit** changes at a time

<i>code</i>	<i>subsets</i>	<i>moves</i>
0 0 0 0	<i>empty</i>	
0 0 0 1	1	enter 1
0 0 1 1	2 1	enter 2
0 0 1 0	2	exit 1
0 1 1 0	3 2	enter 3
0 1 1 1	3 2 1	enter 1
0 1 0 1	3 1	exit 2
0 1 0 0	3	exit 1
1 1 0 0	4 3	enter 4
1 1 0 1	4 3 1	enter 1
1 1 1 1	4 3 2 1	enter 2
1 1 1 0	4 3 2	exit 1
1 0 1 0	4 2	exit 3
1 0 1 1	4 2 1	enter 1
1 0 0 1	4 1	exit 2
1 0 0 0	4	exit 1

Gray code representations

Gray code: solution

<i>code</i>	<i>subsets</i>	<i>moves</i>
0 0 0 0	<i>empty</i>	
0 0 0 1	1	enter 1
0 0 1 1	2 1	enter 2
0 0 1 0	2	exit 1
0 1 1 0	3 2	enter 3
0 1 1 1	3 2 1	enter 1
0 1 0 1	3 1	exit 2
0 1 0 0	3	exit 1
1 1 0 0	4 3	enter 4
1 1 0 1	4 3 1	enter 1
1 1 1 1	4 3 2 1	enter 2
1 1 1 0	4 3 2	exit 1
1 0 1 0	4 2	exit 3
1 0 1 1	4 2 1	enter 1
1 0 0 1	4 1	exit 2
1 0 0 0	4	exit 1

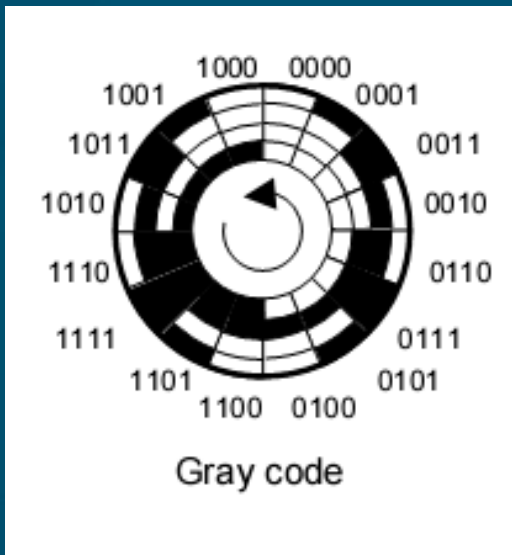
Gray code representations

- Same as **Hanoi!**

```
void gray(int n, bool enter) {  
    if (n > 0) return;  
    gray(n-1, true);  
    move(n, enter);  
    gray(n-1, false);  
}
```

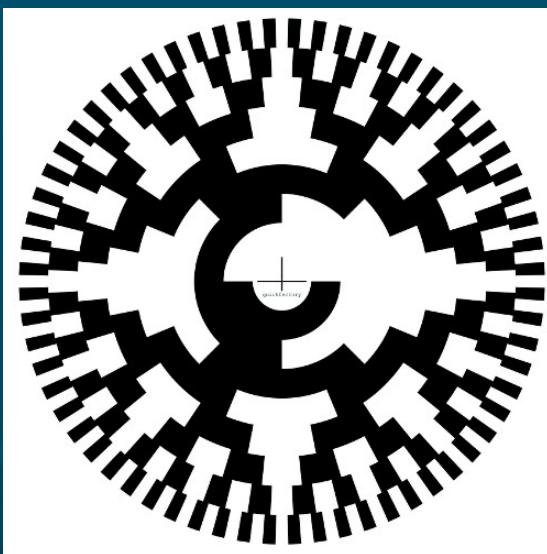
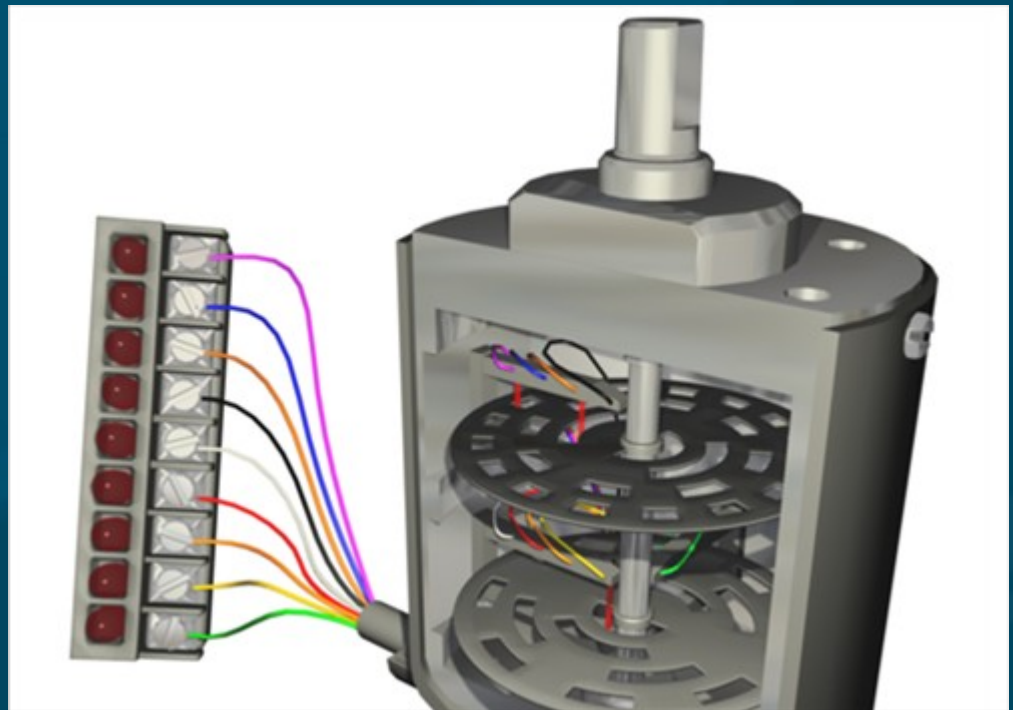
// enter: true => enter,
false => exit

Gray code wheels



4 bits

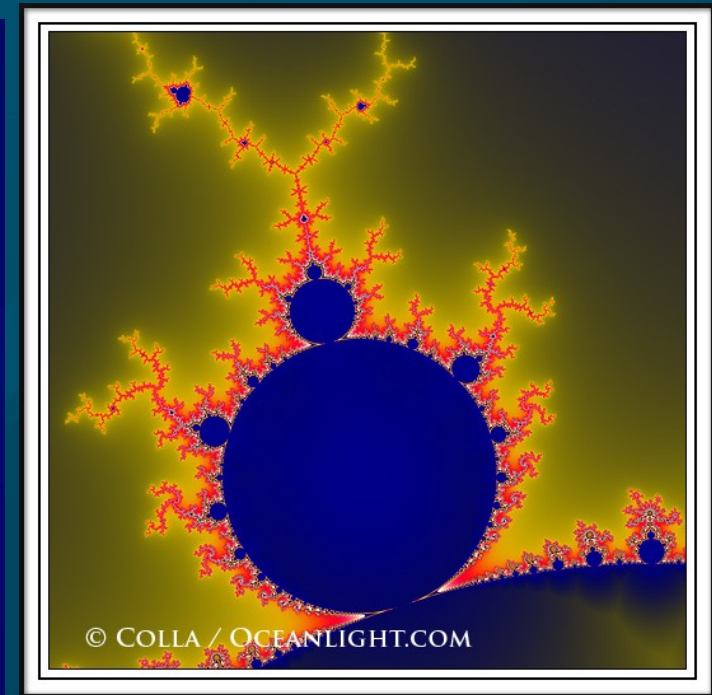
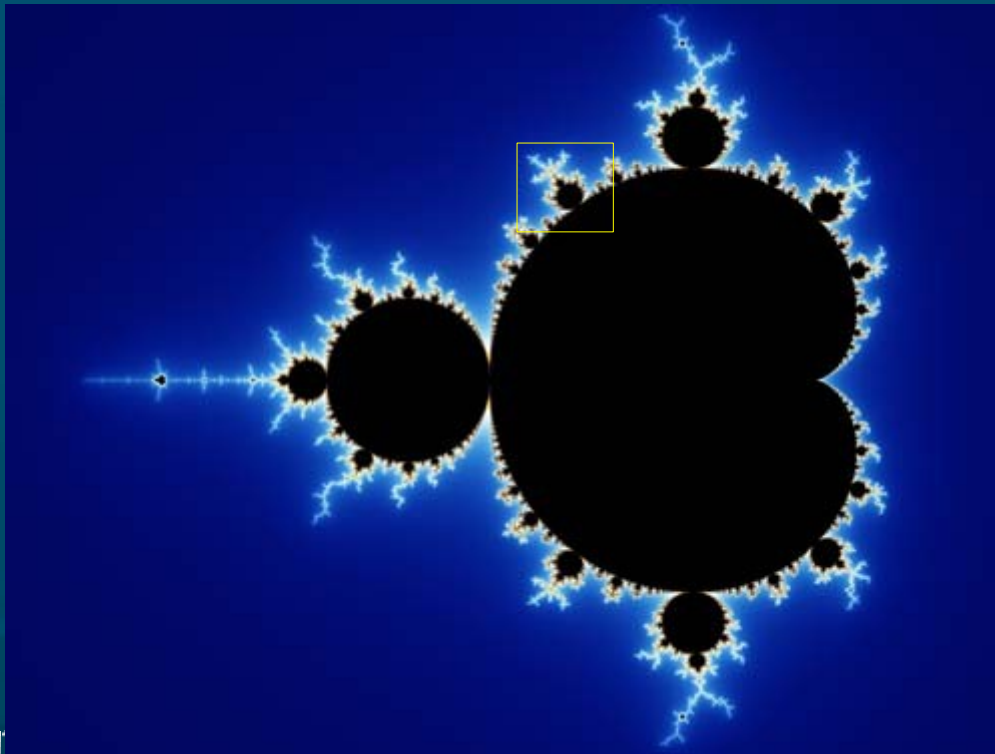
rotary encoding:
incremental rotation
only changes 1 bit
at a time



8 bits

Fractals

- **Fractals** are geometric shapes exhibiting **self-similarity**: zoom in on a portion and it looks just like the original
- **Mandelbrot set**:



Iterated function systems

- IFSs are a kind of fractal generated by recursive geometric replacement rules
- Sierpinski triangle: start with any triangle
 - Cut out the middle triangle: corners are halfway along each side
 - Recurse for each of the three subtriangles



Sierpinski and the Chaos Game

- **Chaos game**: choose **three** points on a plane, colour them say red/green/blue.
- Start at a random **seed** point.
- **Roll** a die and choose red/green/blue.
- Move **halfway** toward that point.
- **Repeat**
- What **pattern** is formed?

