Recursion

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CMPT140
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What's on for today

- Recursion
  - Call stack
  - Application: factorial()
  - Tail recursion (and why it's bad!)
  - Application: fibonacci()

Recursion

- Recursion is when a function invokes itself
- Classic example: factorial (!)
  - n! = n(n-1)(n-2)(n-3) … (3)(2)(1)
  - 0! = 1
- Compute recursively:
  - Inductive step: n! = n*(n-1)!
  - Base case: 0! = 1
- Inductive step: assume (n-1)! is calculated correctly; then we can find n!
- Base case is needed to tell us where to start

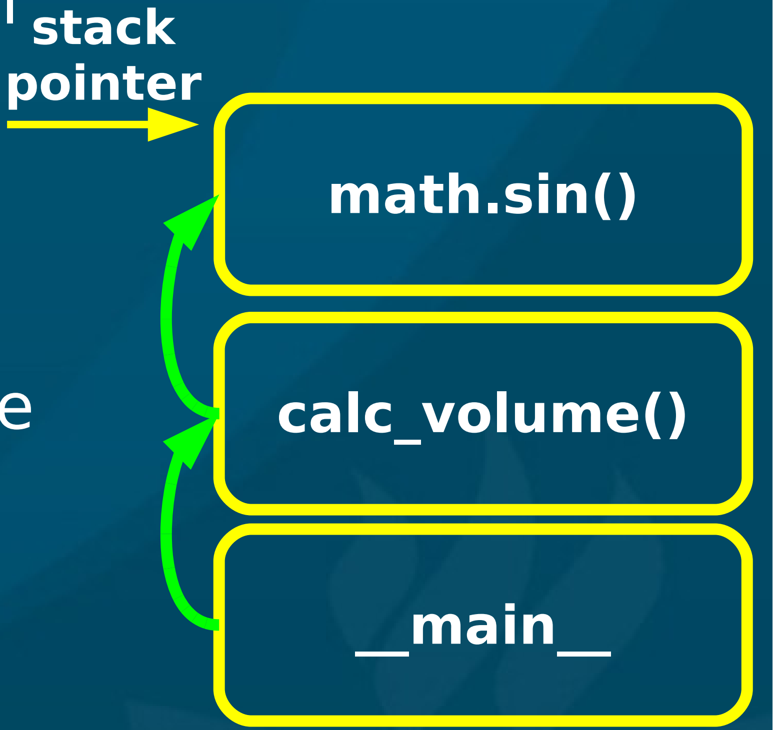
factorial() in Python

```
def factorial(n):
    """Calculate n!.  n should be a positive
      integer."""
    if n == 0:                          # base case
        return 1
    else:                               # inductive step
        return n * factorial(n-1)
```

- Progress is made each time: factorial(n-1)
- Base case prevents infinite recursion
- What about factorial(-1)? Or factorial(2.5)?

The call stack

- When a program is running, an area of memory is set aside to store local variables, the state of the program, etc.
- When a procedure is invoked, the calling context is saved, and a new chunk of memory is allocated for the procedure to use: its stack frame
- When the procedure finishes, its frame is released and control goes back to the calling context
- The stack pointer keeps track of what frame is currently running



Call stack for recursion

```
def factorial(n):
    """Compute the factorial of a
      positive integer."""
    if n == 0:
        return 1
    else:
        return n*factorial(n-1)
```

- If there were any local variables, each frame would have its own instance of the local variables
- When an error (exception) happens, IDLE shows a backtrace: part of the call stack

Tail recursion is bad

- Because the Python runtime must keep all the stack frames from previous invocations, recursion is relatively slow and uses memory.
- Tail recursion is when the last computation before returning is a recursive call
  - Better to re-write this using loops!

```
def factorial(n):
    prod = 1
    for factor in range(1, n):
        prod *= factor
    return prod
```

Recursion example: Fibonacci

- Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34,..
  - Each number is the sum of the two previous

```
def fibonacci(n):
    """Compute the n-th Fibonnaci number.
    pre: n should be a positive integer."""
    if n == 0 or n == 1:            # base case
        return 1
    else:                           # inductive step
        return fibonacci(n-2) + fibonacci(n-1)
```

  - Note: very inefficient algorithm!

Python type hierarchy (partial)

- Atomic types
  - Numbers
    - Integers (int, long, bool): 5, 500000L, True
    - Reals (float) (only double-precision): 5.0
    - Complex numbers (complex): 5+2j
- Container (aggregate) types
  - Immutable sequences
    - Strings (str): "Hello"
    - Tuples (tuple): (2, 5.0, "hi")
  - Mutable sequences
    - Lists (list): [2, 5.0, "hi"]
  - Mappings
    - Dictionaries (dict): {"apple": 5, "orange": 8}

Dictionaries

- Python dictionaries are mutable, unsorted containers holding associative key-value pairs
- Create a dictionary with curly braces {}:
  - **appleInv = {'Fuji': 10, 'Gala': 5, 'Spartan': 7}**
- Index a dictionary using a key:
  - **appleInv['Fuji']   # returns 10**
- Values can be any object and may mix types:
  - **appleInv['Rome'] = range(3)**
- Keys can be any immutable type (even tuples!):
  - **appleInv[('BC', 'Red Delicious')] = 12**