

**Trinity Western University**  
**Department of Mathematical Sciences**  
**MATH 250 (Linear Algebra)**  
**Sample Mid-Term Exam I**

1. Show that the matrix

$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$$

satisfies the equation

$$(A + 3I)^2(A - I) = 0$$

Use the above equation to prove that  $A$  is invertible and compute  $A^{-1}$ .

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 5 \\ 1 & -7 & 13 \end{pmatrix}$$

Show that  $A$  is not invertible by finding a lower-triangular matrix  $L$  such that  $A = LU$ , where  $U$  is an upper-triangular matrix which has at least one row of zeros.

3. Find for what values of  $c$  the following matrix is not invertible. Find the inverse of the matrix for the remaining values of  $c$ .

$$\begin{pmatrix} 1 & 0 & -c \\ -1 & 3 & 1 \\ 0 & 2c & -4 \end{pmatrix}$$

4. Using Cramer's rule solve the following system of equations for  $z$ :

$$\begin{aligned} x + y + z + w &= 10 \\ x + 2y + 3z + 4w &= 30 \\ x + 4y + 9z + 16w &= 100 \\ x + 8y + 27z + 64w &= 354 \end{aligned}$$

5. Assume that there are three classes – upper  $U$ , middle  $M$ , and lower  $L$  – and that social mobility is modeled as follows:

i) Of children of  $U$  parents, 70% remain  $U$  while 20% become  $L$  and 10% become  $M$ .

ii) Of children of  $M$  parents, 80% remain  $M$  while 10% become  $L$  and 10% become  $U$ .

iii) Of children of  $L$  parents, 60% remain  $L$  while 10% become  $U$  and 30% become  $M$ .

Find the probability that the grandchild of  $L$  parents becomes  $U$ . Also find the long-term breakdown of society into classes.