

**Trinity Western University**  
**Department of Mathematical Sciences**  
**MATH250 (Linear Algebra)**  
**Sample Mid-Term II Examination**

1. Let A and B be the end points of a diameter of a circle. If C is any other point on the circle, show that AC and BC are perpendicular.
2. Find the equation of the line passing through  $P_0(1, 1, 2)$  intersecting the line  $L: (x, y, z) = (2, 1, 0) + t(1, 1, 1)$ , and perpendicular to that line.
3. For what value(s) of  $k$  and  $(w_1, w_2, w_3)$  the range of the linear operator defined by the equations
$$\begin{aligned}w_1 &= x_1 + 2x_2 + x_3 \\w_2 &= -2x_1 + x_2 + 4x_3 \\w_3 &= 7x_1 + 4x_2 + kx_3\end{aligned}$$
is not in  $\mathbb{R}^3$ ?  
Also for any value of  $k$ , find which vectors  $(x_1, x_2, x_3)$  map into the line  $w_1 = 1+2t$ ,  $w_2 = 1+t$ ,  $w_3 = 1+4t$ .
4. If  $V$  is a set of ordered pairs  $(x, y)$  of real numbers with the following operations.  
 $(x, y) + (x', y') = (x + x', y + y' + 1)$  and  $k(x, y) = (kx, ky + k - 1)$ ,  
determine if it is a vector space. If it is not, list all axioms that fail to hold.
5. Is the set  $V$  of all  $2 \times 2$  matrices with equal column sums a subspace of  $M_{22}$ ? If not, why not?