Ch4: Probability

- **HW2 due Thu 10pm**

20 Sep 2011
BUSI275
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Outline for today

- Empirical Rule and z-scores
- Probability: events, outcomes, sample space
  - Event trees
- Venn diagrams and set theory:
  - Complement, union, intersection
- Addition rule for A ∪ B
  - Mutual exclusivity
- Conditional probability and Bayes' rule
- Statistical independence

z-scores

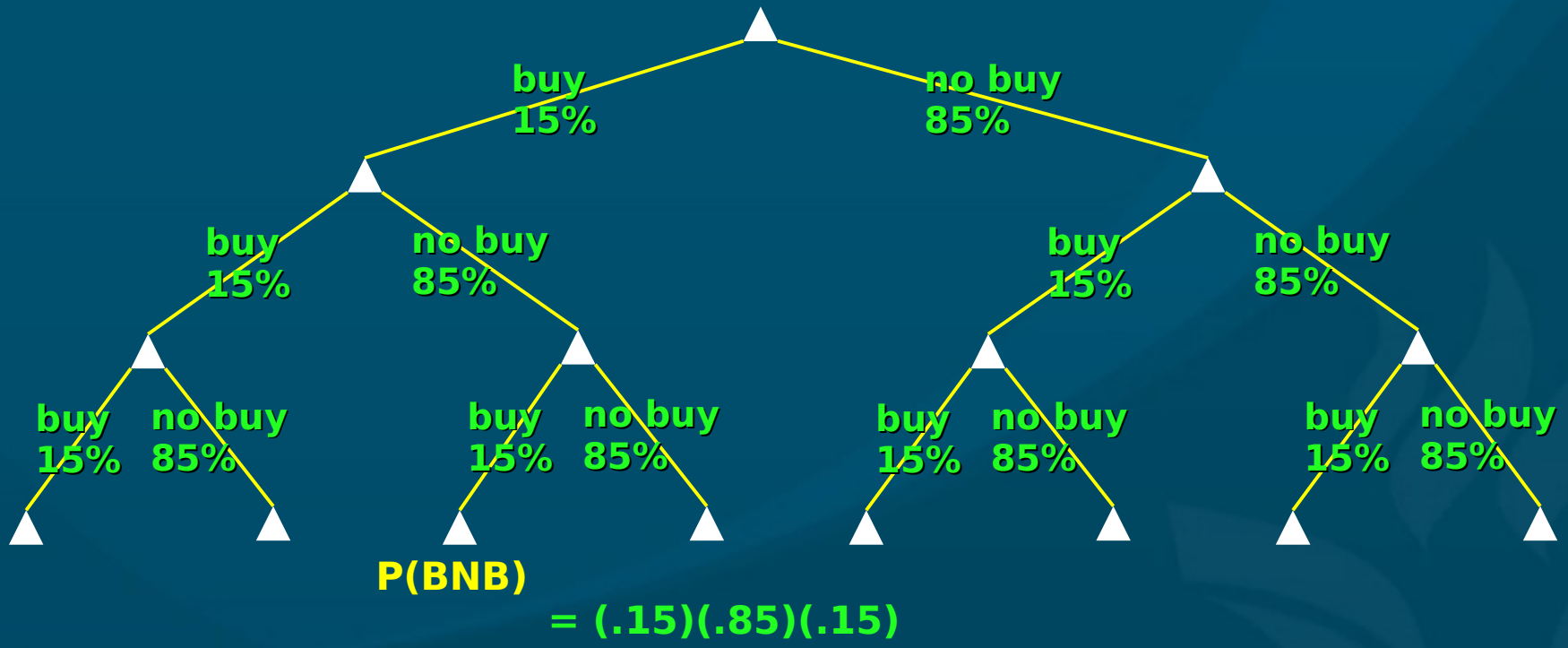
- Describes a value's position relative to the mean, in units of standard deviations:
  - $z = (x - \mu)/\sigma$
- e.g., you got a score of 35 on a test: is this good or bad? Depends on the mean, SD:
  - $\mu=30$, $\sigma=10$: then $z = +0.5$: pretty good
  - $\mu=50$, $\sigma=5$: then $z = -3$: really bad!

Probability

- Chance of a particular event happening
- e.g., in a sample of 1000 people, say 150 will buy your product:
  - ⇒ the probability that a random person from the sample will buy your product is 15%
  - Experiment: pick a random person (1 trial)
  - Possible outcomes: {“buy”, “no buy”}
  - Sample space: {“buy”, “no buy”}
  - Event of interest: A = {“buy”}
  - P(A) = 15%

Event trees

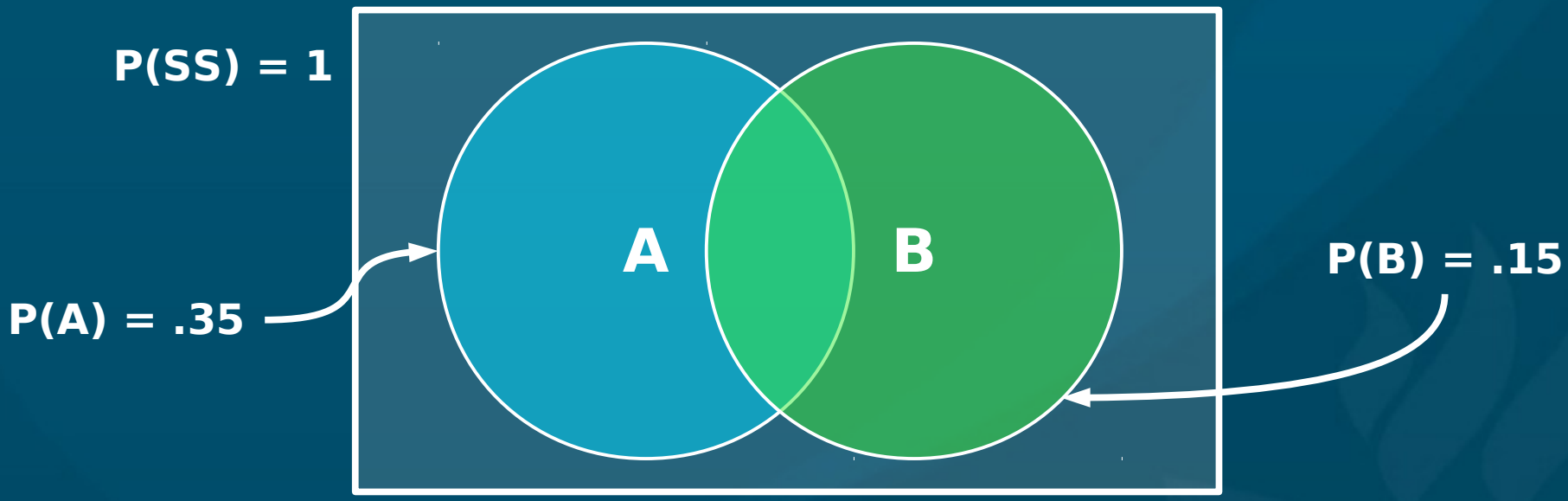
- Experiment: pick 3 people from the group
- Outcomes for a single trial: {“buy”, “no buy”}
- Sample space: {BBB, BBN, BNB, BNN, NBB, …}



- Event: A = {at least 2 people buy}: P(A) = ?

Venn diagrams

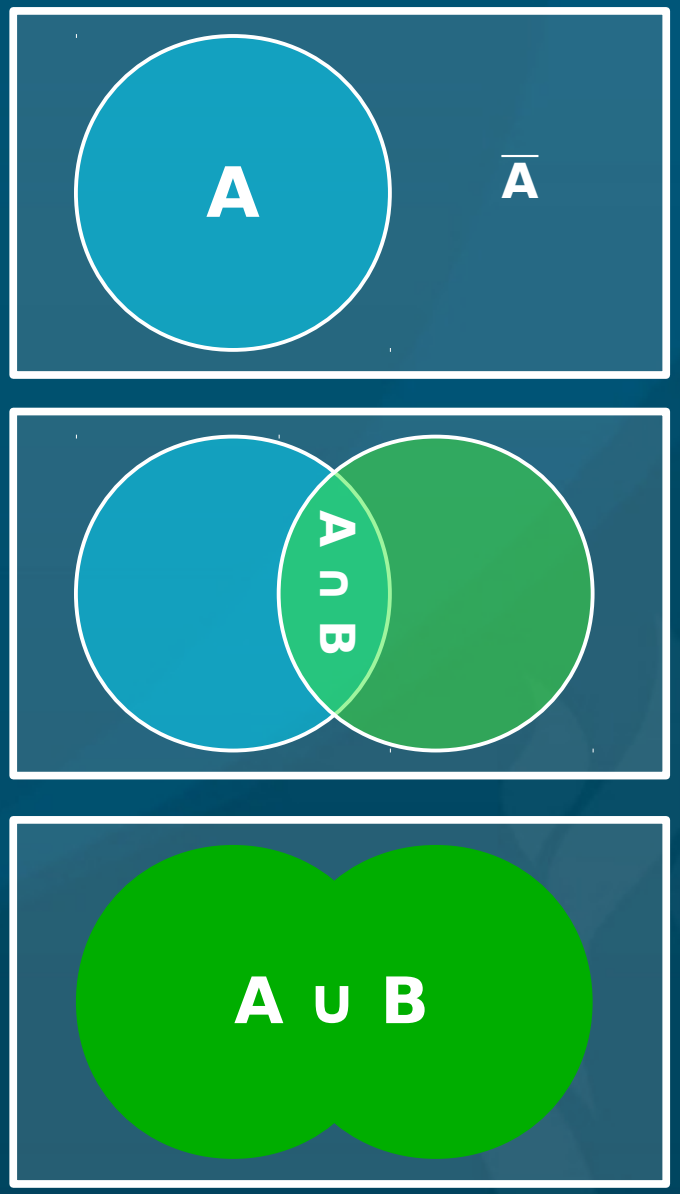
- Box represents whole sample space
- Circles represent events (subsets) within SS
- e.g., for a single trial:



  - A = "clicks on ad"
  - B = "buys product"

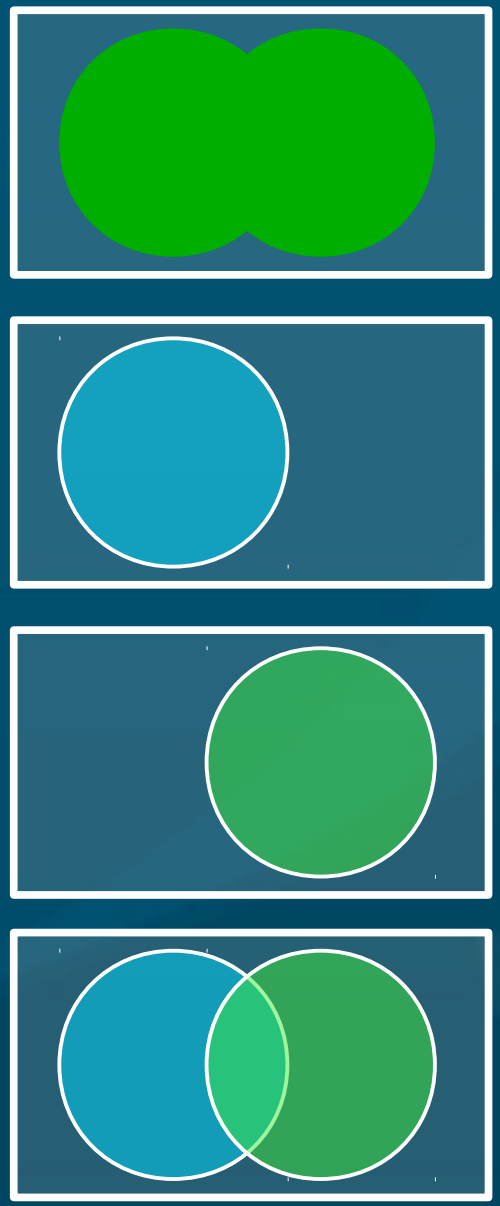
Venn: set theory

- Complement: $\overline{A}$
  = “does not click ad”
  - $P(\overline{A}) = 1 - P(A)$
- Intersection: $A \cap B$
  = “clicks ad and buys”
- Union: $A \cup B$
  = “either clicks ad or buys”



Addition rule: A ∪ B

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Addition rule: example

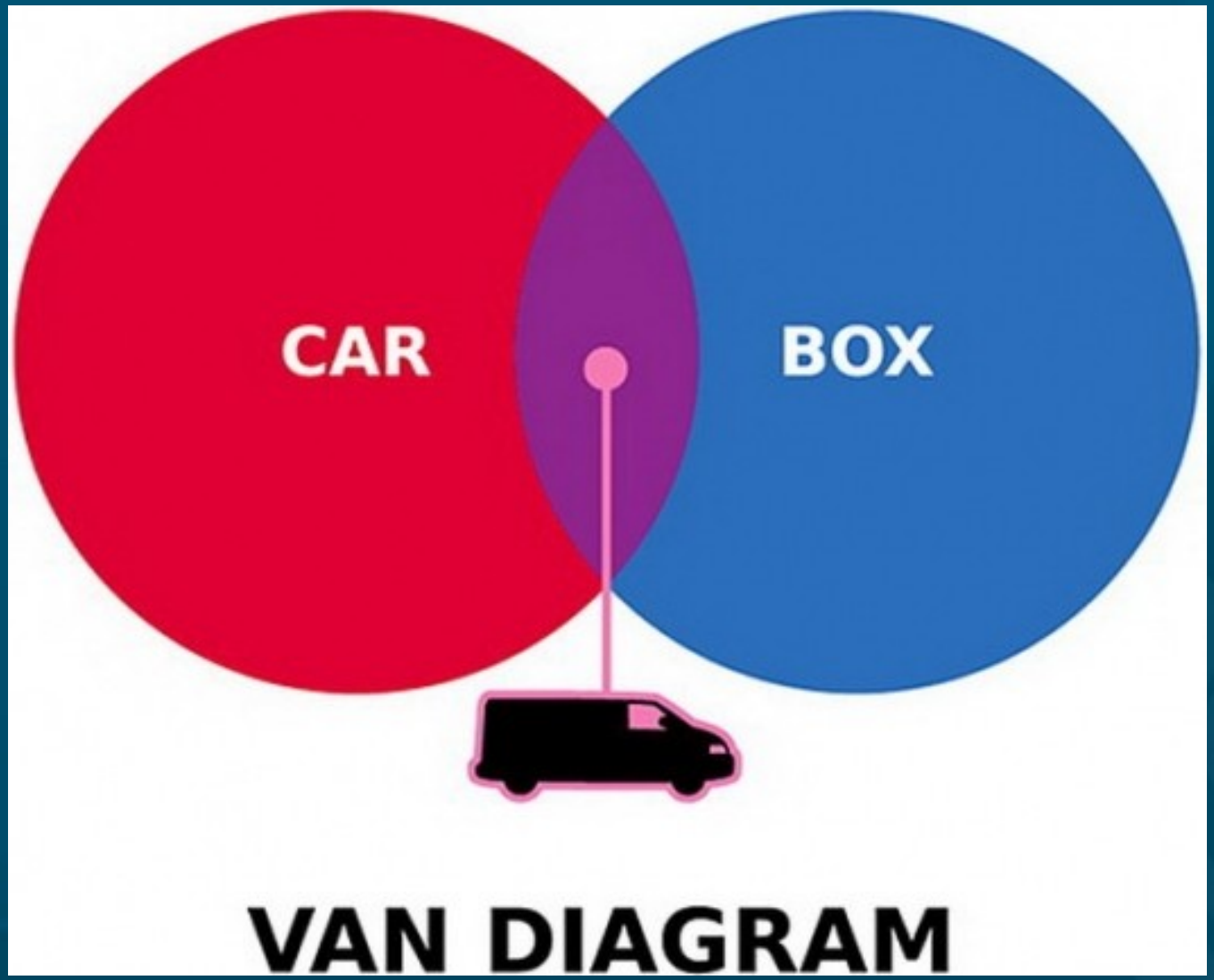
- 35% of the focus group clicks on ad:
  - P(?) = .35
- 15% of the group buys product:
  - P(?) = .15
- 45% are "engaged" with the company: either click ad or buy product:
  - P(?) = .45
- ⇒ What fraction of the focus group buys the product through the ad?
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
    ? = ? + ? - ?

Mutual exclusivity

- Two events A and B are mutually exclusive if the intersection is null: $P(A \cap B) = 0$
  - i.e., an outcome cannot satisfy both A and B simultaneously
- e.g., A = male, B = female
- e.g., A = born in Alberta, B = born in BC

- If A and B are mutually exclusive, then the addition rule simplifies to:
  - $P(A \cup B) = P(A) + P(B)$

Yep!



Conditional probability

- P(A|B): probability of A given B
  - "Narrows" the sample space to B
- P(buy) might be pretty small
  - Especially if nobody's heard of us
- P(buy | likes ad) could be much bigger
  - If the ad is effective: conversion rate

- Bayes' Theorem (rule): $P(A|B) = P(A \cap B) / P(B)$

Bayes' Theorem: example

- Let: A = likes our ad
  B = buys our product
- 40% of the focus group likes our ad
  - P(?) = .40
- Of those who like our ad, 10% buy our product (i.e., 10% conversion rate)
  - P(?) = .10
- ⇒ What fraction of the focus group buys our product?
  - P(B | A) = P(A ∩ B) / P(A)
    ? = ? / ?

Statistical independence

- Two events A and B are independent when:
  - P(A|B) = P(A), or equivalently, P(B|A) = P(B)
  - One event being true does not change the probability of the other event happening
- e.g., A = wears socks, B = has blue eyes
  - P(wearing socks) is the same regardless of whether the person has blue eyes
- e.g., A = first person buys our product, B = next (random) person buys our product
  - Assumes customers don't talk to each other
- Would these be independent?
  - A = likes our ad, B = buys our product

Indep. vs. mutual exclusivity

- A = likes our ad, B = buys our product
- But what if someone says:
  - "Just because someone doesn't like our ad, doesn't mean that they can't still buy our product – just because event A is true doesn't mean that event B is impossible."
- Is this a statement about independence or mutual exclusivity?

Independence and Bayes' rule

- Recall Bayes: $P(A|B) = P(A \cap B) / P(B)$
- Rewrite this as:
  - $P(A \cap B) = P(A|B)\, P(B)$
- Now, if we also know that A and B are statistically independent, then $P(A|B) = P(A)$, so
  - $P(A \cap B) = P(A)\, P(B)$
- P(both customers buy) = P(cust1 buys) P(cust2 buys)

TODO

- HW2 (ch2-3): due Thu 22 at 10pm
  - Remember to format as a document!
  - HWs are to be individual work
- Get to know your classmates and form teams
  - Email me when you know your team
  - You can come up with a good name, too
- Discuss topics/variables you are interested in
  - Find existing data, or gather your own?