Ch6: Continuous Distributions

- **HW3 due Thu 10pm**
- **Download and open: 06-Normal.xls**

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BUSI275
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TRINITY WESTERN UNIVERSITY

Outline for today

- Hypergeometric distribution: HYPGEOMDIST()
- Normal distribution
  - NORMDIST()
  - Cumulative normal
  - Continuity correction
  - Standard normal
- Uniform distribution
  - μ and σ
- Exponential distribution: EXPONDIST()
  - Density and cumulative

Hypergeometric distribution

- n trials taken from a finite population of size N
- Trials are drawn without replacement: the trials are not independent of each other
  - Probabilities change with each trial
- Given that there are X successes in the larger population of size N, what is the chance of finding exactly x successes in these n trials?

$$P(x) = \frac{\binom{X}{x}\binom{N-X}{n-x}}{\binom{N}{n}} \qquad \left(\text{recall } \binom{n}{x} = \frac{n!}{x!(n-x)!}\right)$$

Hypergeometric: example

- In a batch of 10 lightbulbs, 4 are defective.
- If we select 3 bulbs from that batch, what is the probability that 2 out of the 3 are defective?
  - Population: N=10, X=4
  - Sample (trials): n=3, x=2

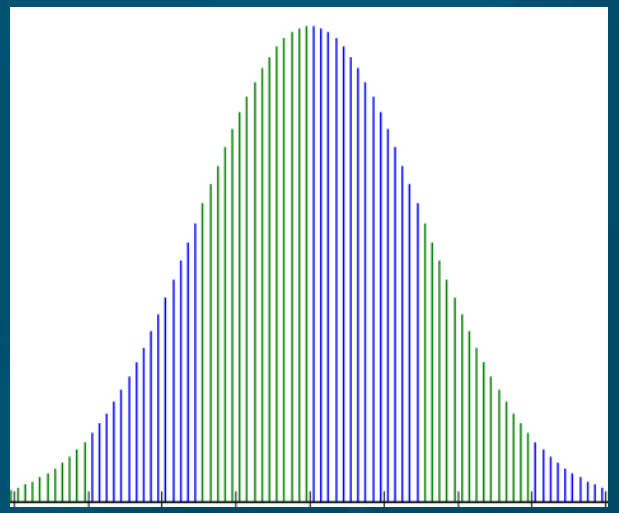
$$P(2) = \frac{\binom{4}{2}\binom{10-4}{3-2}}{\binom{10}{3}} = \frac{\left(\frac{4!}{2*2}\right)\left(\frac{6!}{1*5!}\right)}{\left(\frac{10!}{3!*7!}\right)} = \frac{(3!)(6)}{\left(\frac{10*9*8}{3!}\right)} = \frac{3}{10}$$

- In Excel: HYPGEOMDIST(x, n, X, N)
  - HYPGEOMDIST(2, 3, 4, 10) → 30%

Normal distribution

- The normal "bell" curve has a formal definition:

$$N(\mu,\sigma)(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



- Mean is μ, standard deviation is σ
- Drops exponentially with z-score
- Normalized so total area under curve is 1
- Excel: NORMDIST(x, μ, σ, cum)
  - e.g., exam has μ=70, σ=10.
    What is probability of getting a 65?
  - =NORMDIST(65, 70, 10, 0) → 3.52%

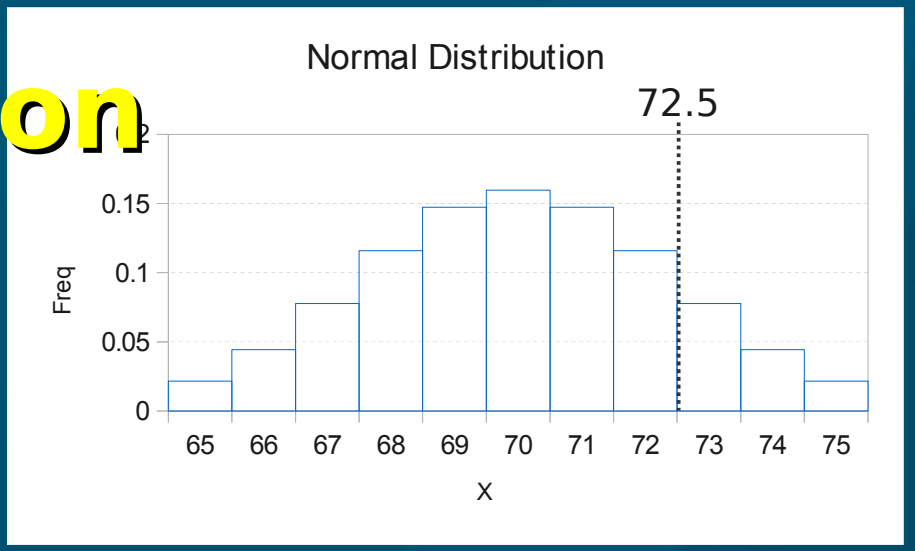
Cumulative normal

- Usually, we are interested in the probability over a range of values:
  - Area of a region under the normal curve
- The cumulative normal gives area under the normal curve, to the left of a threshold:
  - e.g., exam with $\mu=70$, $\sigma=10$.
    What is probability of getting below 65?
  - =NORMDIST(65, 70, 10, 1) → 30.85%
  - e.g., getting between 75 and 90?
  - =NORMDIST(90, 70, 10, 1) –
    NORMDIST(75, 70, 10, 1) → 28.58%

Inverse function

- Excel can also find the threshold (x) that matches a given cumulative normal probability:
  - NORMINV(area, μ, σ)
- E.g., assume air fares for a certain itinerary are normally distrib with σ=$50 but unknown μ.
  The 90th percentile fare is at $630.
  What is the mean air fare?
  - We have: NORMINV(.90, μ, 50) = 630, so
  - =630 – NORMINV(.90, 0, 50) → μ=$565.92

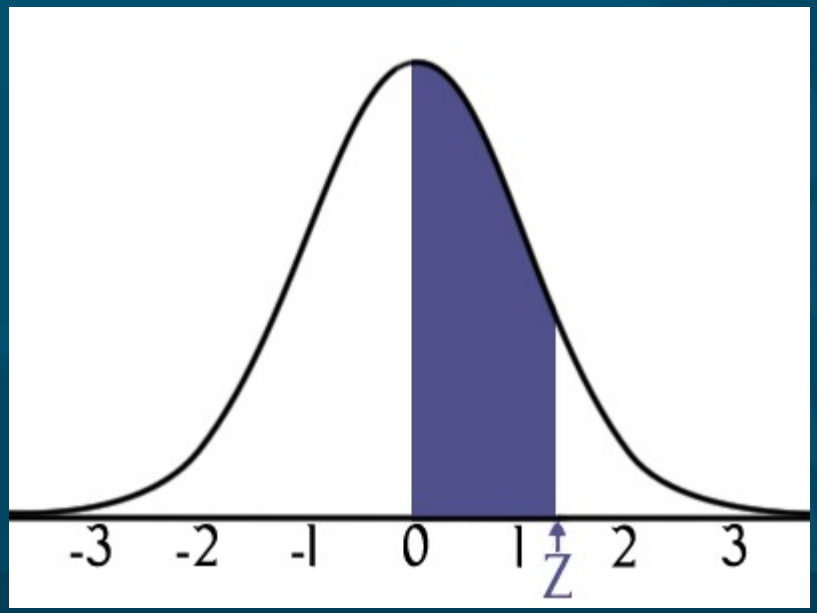
Continuity correction



- For discrete variables (e.g., integer-valued):
  - e.g., # of students per class, assumed to be normally distributed with $\mu=25$, $\sigma=10$
- The range can be inclusive or exclusive:
  - Probability of a class having fewer than 10?
    - <10: excludes 10
  - At least 30 students? ≥ 30: includes 30
- Edge of the bar is at ± 0.5 from the centre
  - <10: =NORMDIST(9.5, 25, 10, 1) → 6.06%
  - ≥ 30: =1-NORMDIST(29.5, 25, 10, 1) → 32.6%

Standard normal

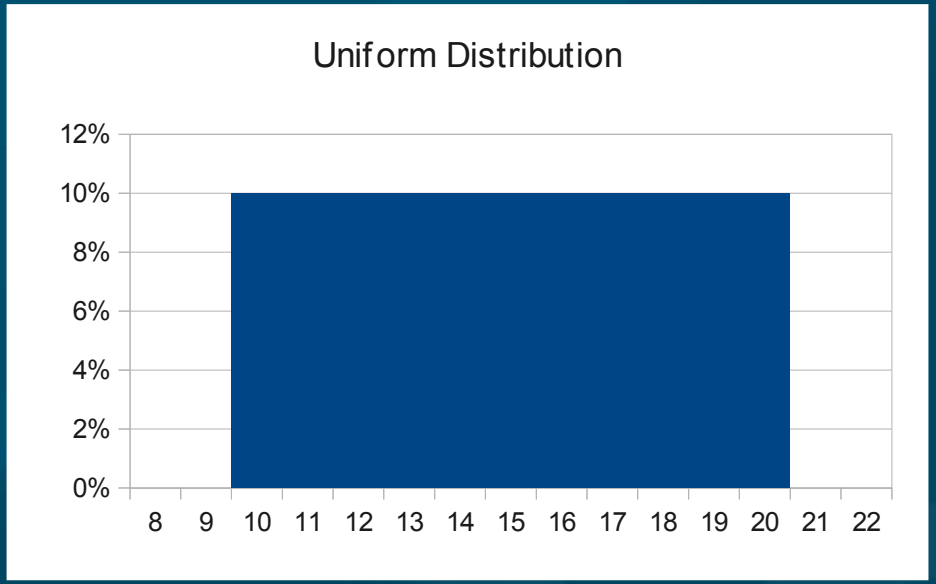
- There is a whole family of normal distributions, with varying means and standard deviations
- The standard normal is the one that has $\mu=0$, $\sigma=1$
- This means z-scores and x-values are the same!
- In Excel: NORMSDIST(x) (cumulative only) and NORMSINV(area)



Uniform distribution

- With a uniform distribution, all values within a range are equally likely
  - e.g., roll of a fair die: {1,2,3,4,5,6} all have probability of 1/6
  - Range is from a to b:

$$U(x) = \begin{cases} \frac{1}{b-a} & if\ a \leqslant x \leqslant b \\ 0 & otherwise \end{cases}$$



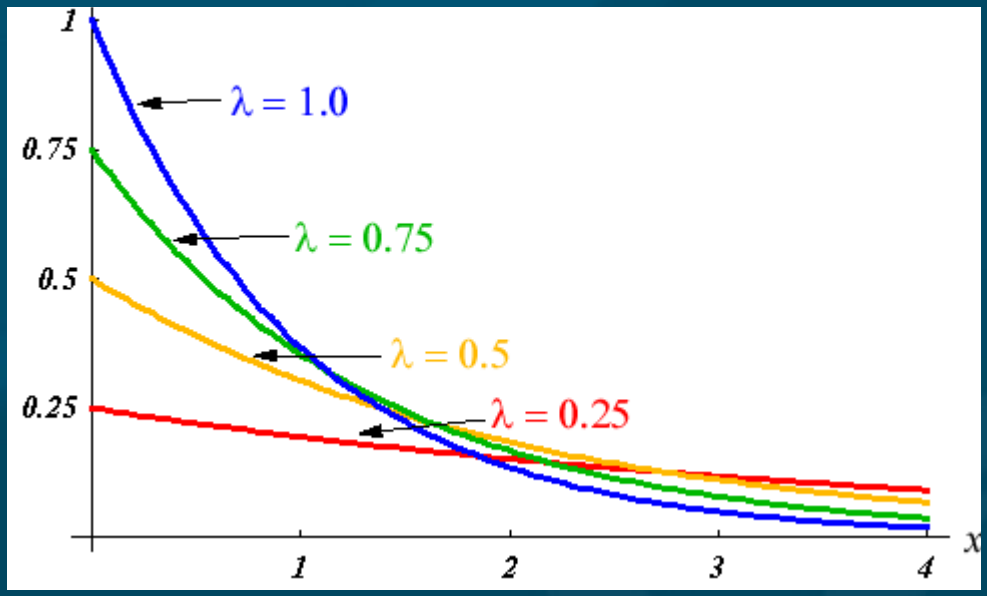
- $\mu=(a+b)/2$, $\sigma=\sqrt{(b-a)^2/12}$

Exponential distribution

- Time between occurrences of an event
  - e.g., time between two security breaches
- Exponential density: probability that the time between occurrences is exactly x is:

$$E(x) = \lambda e^{-\lambda x}$$

  - 1/λ = mean time between occurrences
  - Need both x, λ > 0
- EXPONDIST(x, λ, cum)
  - Density: cum=0



Exponential probability

- Exponential probability (cumulative distribution) is the probability that the time between occurrences is less than x:

$$P(0 \le x \le a) = 1 - e^{-\lambda a}$$

  - Excel: EXPONDIST(x, λ, 1)
- e.g., average time between purchases is 10min. What is the probability that two purchases are made less than 5min apart?
  - EXPONDIST(5, 1/10, 1) → 39.35%
  - Don't forget to convert from 1/λ to λ

REB forms

- Any research involving human subjects requires approval by TWU's Research Ethics Board
- Use of existing public data (e.g., StatCan) does not require REB approval, however:
  - For class purposes, I will still require you to complete an REB form
- Forms are on TWU's REB page
  - Bottom of page: "Request for Ethical Review" or "Analysis of Existing Data"
- Upload to myCourses by 11Oct
  - For non-public data, submit signed paper copy to me by 11Oct

TODO

- HW3 (ch3-4): due Thu at 10pm
  - Remember to format as a document!
  - HWs are to be individual work
- Dataset description due next week! Tue 4 Oct